Stochastic Models for Bubble Creation

and

Bubble Detection Signal Processing Strategies

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Exploratory Bubble Voltage Analysis Strategy

1. Excerpt 1 minute chunks of representative current noise and bubbleogram data for each of several reactor current levels

2. Present the data for examination and comparison

3. Present descriptive statistics of the data for examination and comparison

4. Present the Power Spectral Density function of the data for examination and comparison

5. Present the Autocorrelation function of the data for examination and comparison

6. Present Inter-bubble sojourn time analysis

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General Considerations

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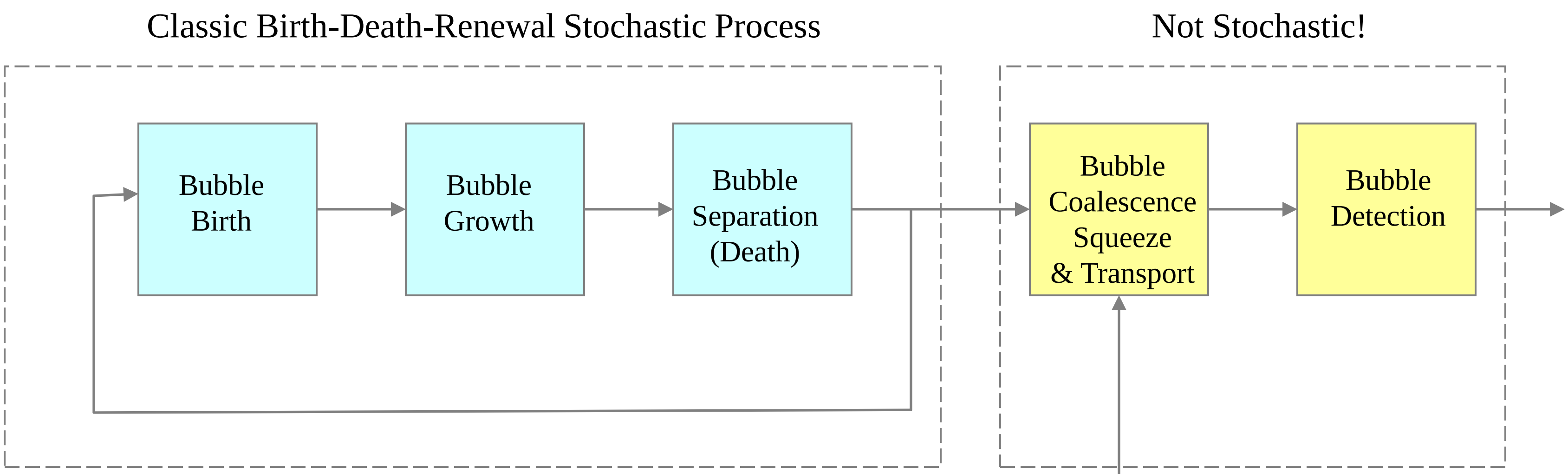
Bubble Detection Stochastic Model



Characterized by three non-observable parameters:

Birth Rate – Growth Rate – Separation Size

Characterized by two unknown parameters:

Coalescence Factor – Squeeze Percent

Observable Parameters:

Cell Flow Rate – Cell Voltage – Cell Current – Cell Pressure – Bubble Det. Voltage

Find: Expected Value of Solute Concentration =

F(Observables) = F(Flow Rate, Voltage, Current, Pressure, Bubble Det. Voltage)

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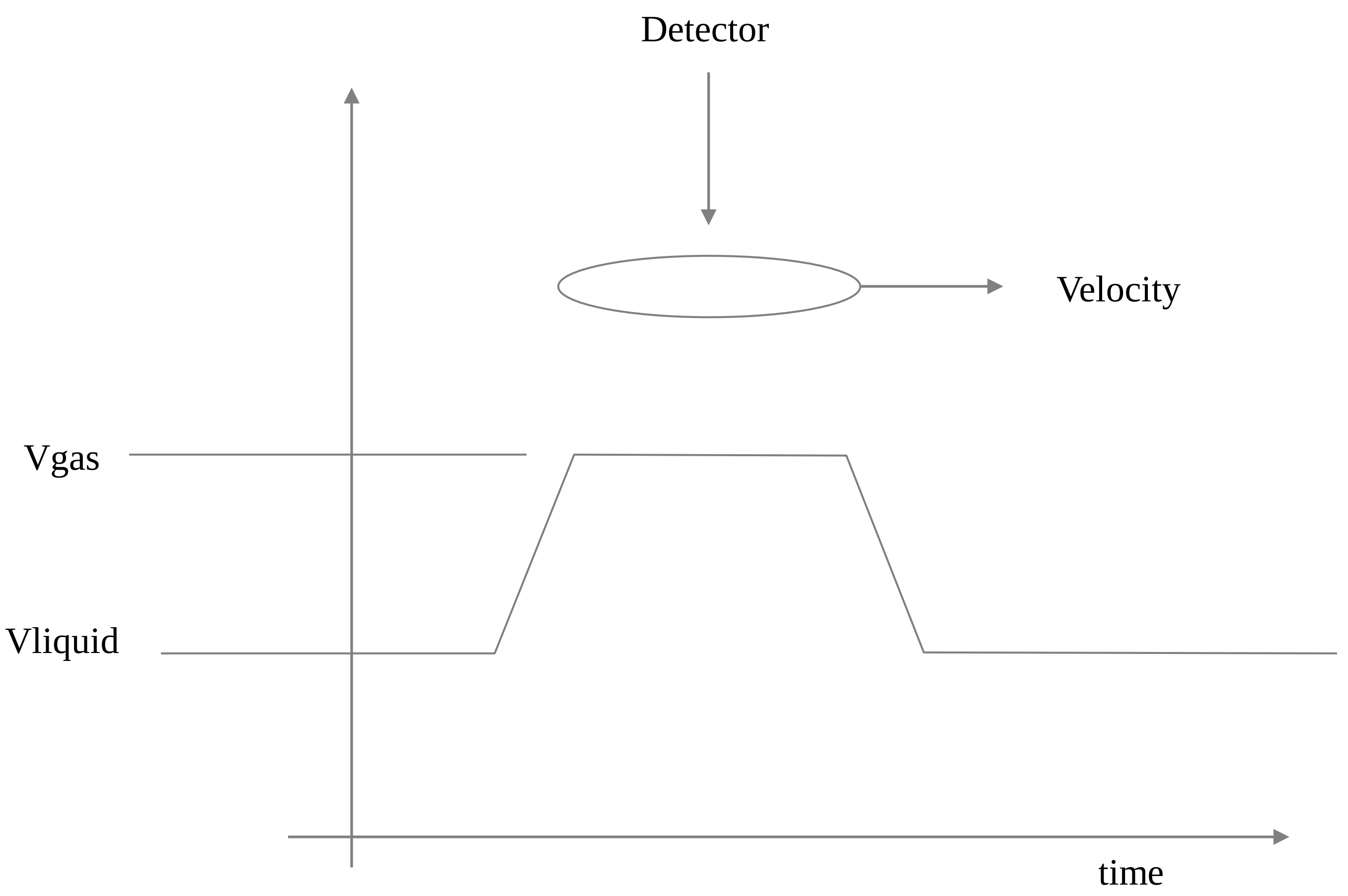
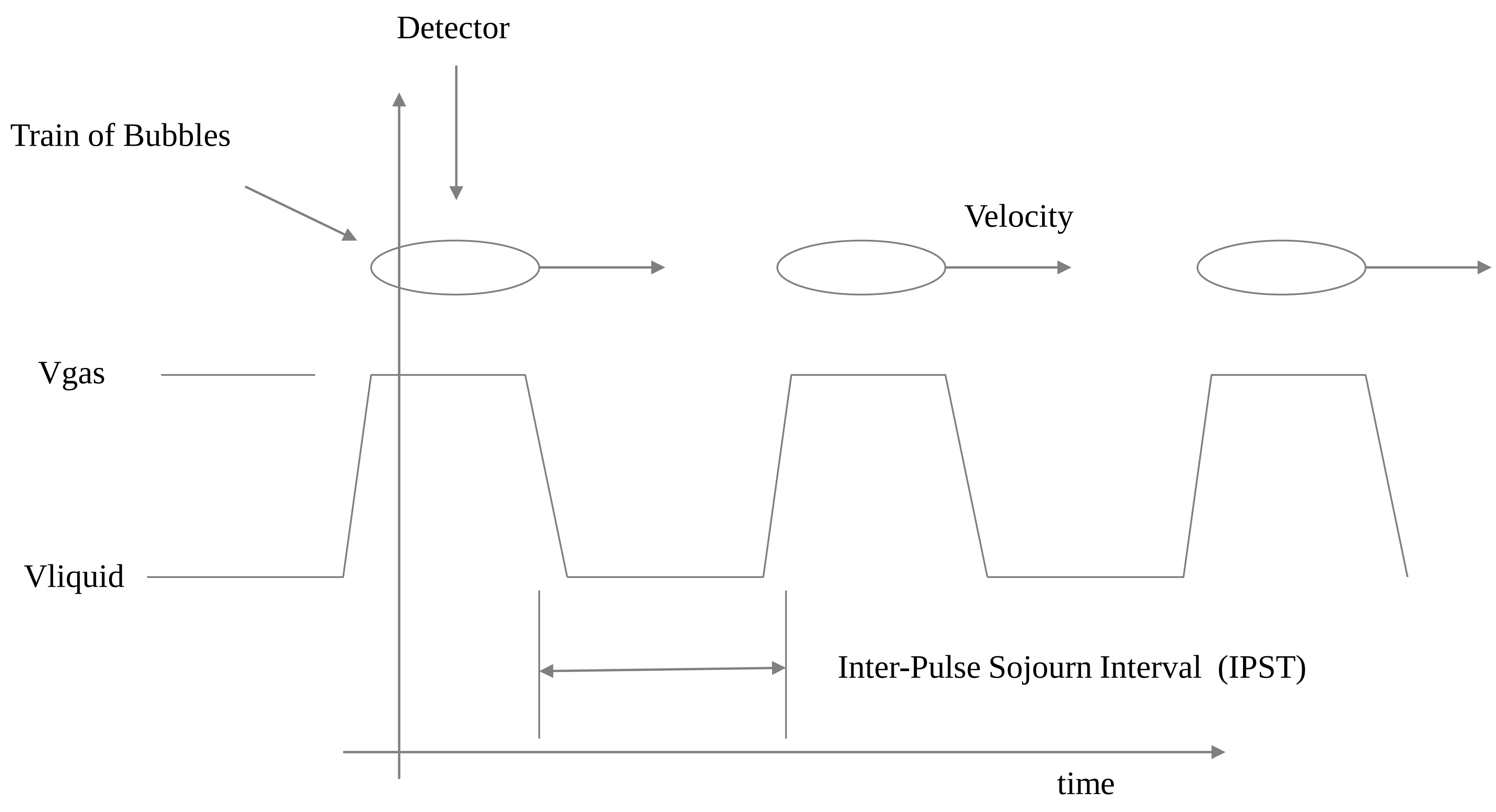
Bubble Detector Response – Single Bubble



Bubble Detector Response – Multiple Bubbles



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Bubble Detector Parameter Extraction Strategies

1. Statistical Methods
    a. Mean
    b. Variance
    c. Range

2. Transform Methods
    a. Power Spectral Density
    b. Autocorrelation

3. Counting Methods
    a. Inter-pulse Sojourn Time
    b. Bubble Duration

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Statistical Methods

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Statistical Methods - Basic

Mean = $\frac{\sum x}{n}$

Standard Deviation = $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

Statistical Methods - Moments

The First Through the Fourth Moments of a Probability Distribution Function

First Moment = the Mean = $\int_{-\infty}^{\infty} x \cdot F(x)\,dx$ "Center of Gravity"

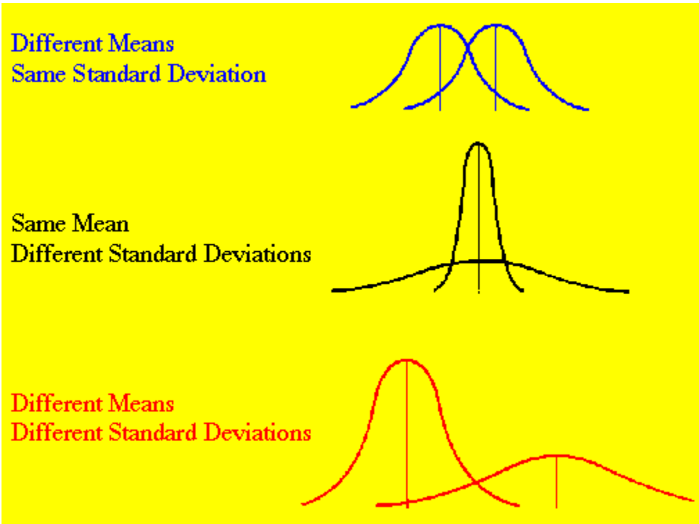
Second Moment = the Variance = $\int_{-\infty}^{\infty} x^2 \cdot F(x)\,dx$ "Radius of Gyration"

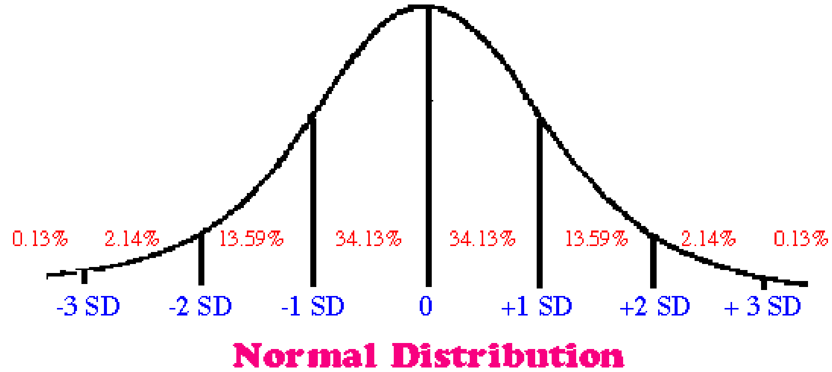
Third Moment = the Skew = $\int_{-\infty}^{\infty} x^3 \cdot F(x)\,dx$ "Measure of Asymmetry"

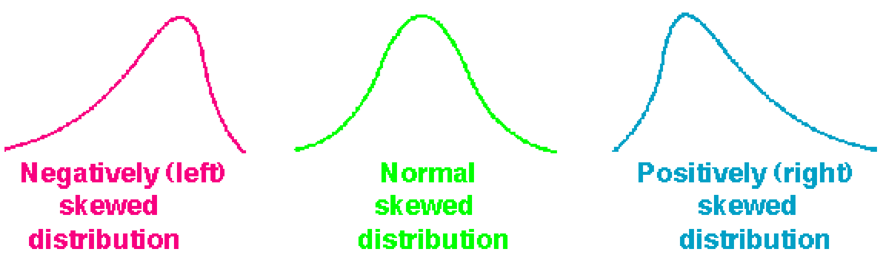
Fourth Moment = the Kurtosis = $\int_{-\infty}^{\infty} x^4 \cdot F(x)\,dx$ "Measure of Central Tendency"

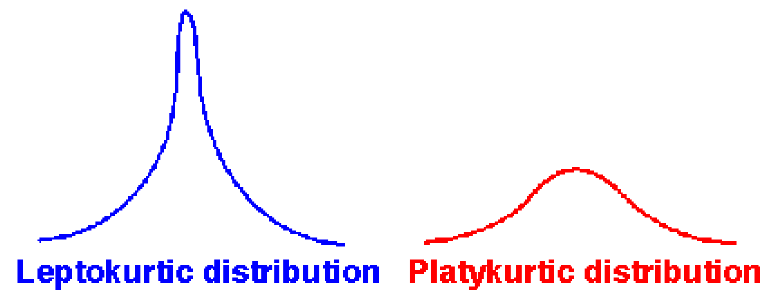
These four parameters quantitatively describe the shape, spread and location of a probability distribution function. Each parameter is the integrated result of all the data in a particular time series and thus may be used to compare the histograms from similar but different fuel cell noise current waveforms. Use of these parameters represents the classical statistical analysis approach to knowledge inference from time series data consisting of information submerged in random data.

Statistical Methods – Moments









Kurtosis - “Measure of skinniness”

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Statistical Models for Bubble Creation and Detection

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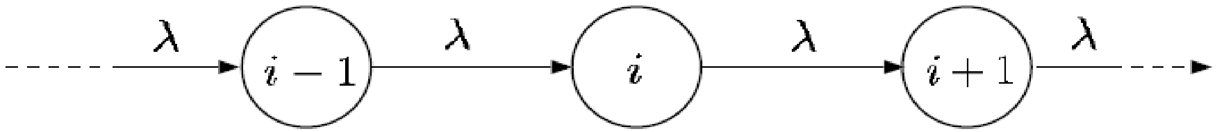
Bubble Oriented Statistical Methods – The Poisson Renewal Process

A stochastic point process $N(t)$ is a Poisson process, if the probability of having one event in any interval $d\,t$ is constant and equal to λ.



Bubble Oriented Statistical Methods – The Poisson Renewal Process

A **Poisson process** is a renewal process in which the inter-arrival times are exponentially distributed with parameter λ.

$$f(t) = \lambda e^{-\lambda t} \quad \Longrightarrow \quad f^*(s) = \frac{\lambda}{s + \lambda}$$

The cdf and density of the time up to the k-th arrival s_k are in LT:

$$f_k^*(s) = \left(\frac{\lambda}{s + \lambda}\right)^k \quad ; \quad F_k^*(s) = \frac{\lambda^k}{s\,(s + \lambda)^k}$$

Let us define:

$$P_k(t) = Pr\{N(t) = k\} = F_k(t) - F_{k+1}(t)$$

Taking Laplace transforms:

$$P_k^*(s) = \frac{\lambda^k}{s\,(s + \lambda)^k} - \frac{\lambda^{k+1}}{s\,(s + \lambda)^{k+1}} = \frac{\lambda^k}{(s + \lambda)^{k+1}}$$

Inverting again in the time domain, we obtain the **Poisson distribution**:

$$P_k(t) = Pr\{N(t) = k\} = \frac{(\lambda\, t)^{\,k}}{k\,!}\, e^{-\lambda\, t}$$

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Bubble Oriented The Poisson Process – Expected # of Events

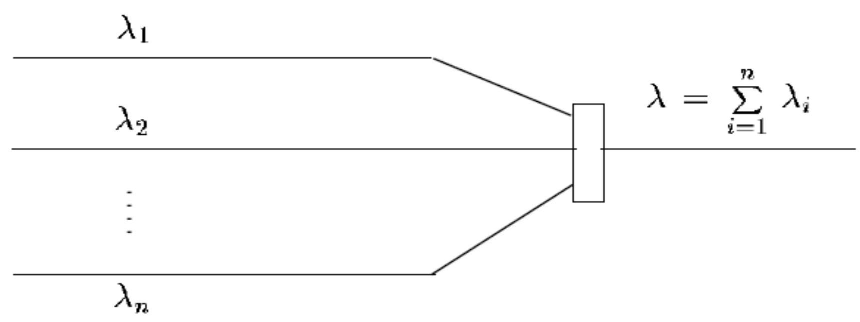
$$H(t) = E[N(t)] = \sum_{k=0}^{\infty} k \, Pr\{N(t) = k\} = \sum_{k=0}^{\infty} k \, P_k(t)$$

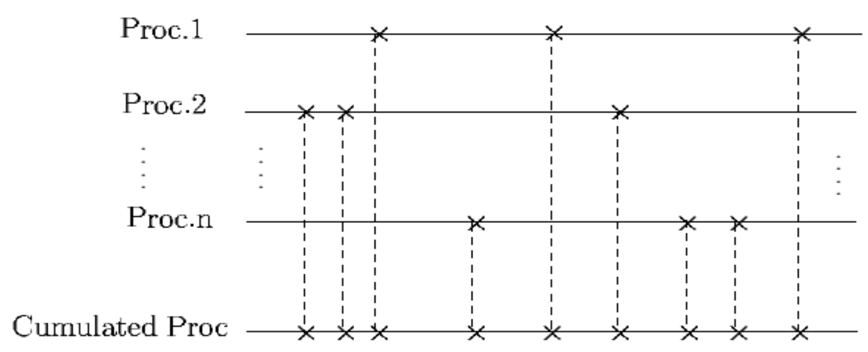
$$= e^{-\lambda t} \sum_{k=1}^{\infty} \frac{(\lambda t)^k}{(k-1)!}$$

$$= \lambda t \cdot e^{-\lambda t} \left(1 + \lambda t + \frac{(\lambda t)^2}{2!} + \cdots \right) = \lambda t$$

The Poisson Process – Superposition

A superposition of Poisson processes is obtained by cumulating the occurrences of n independent sources of Poisson processes with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$, respectively.





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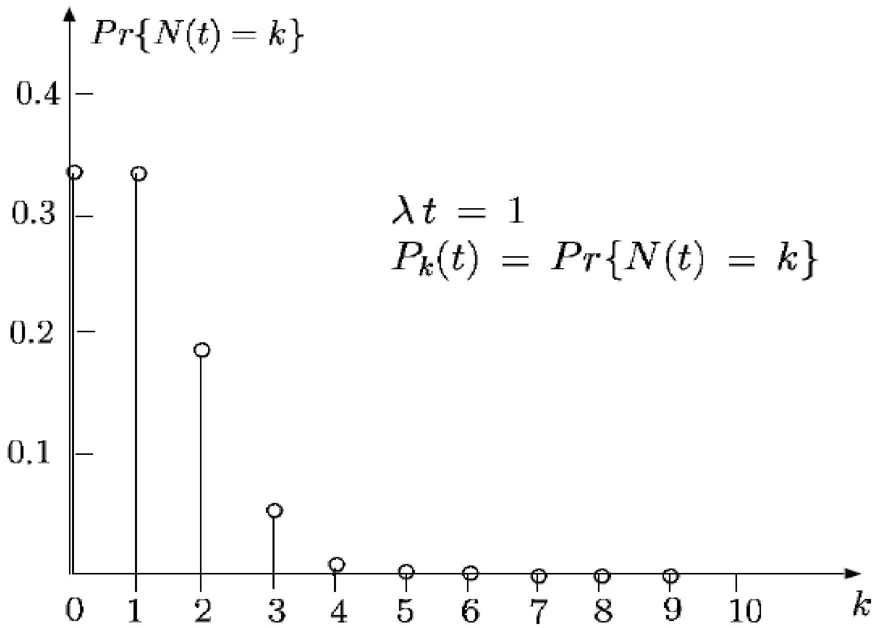
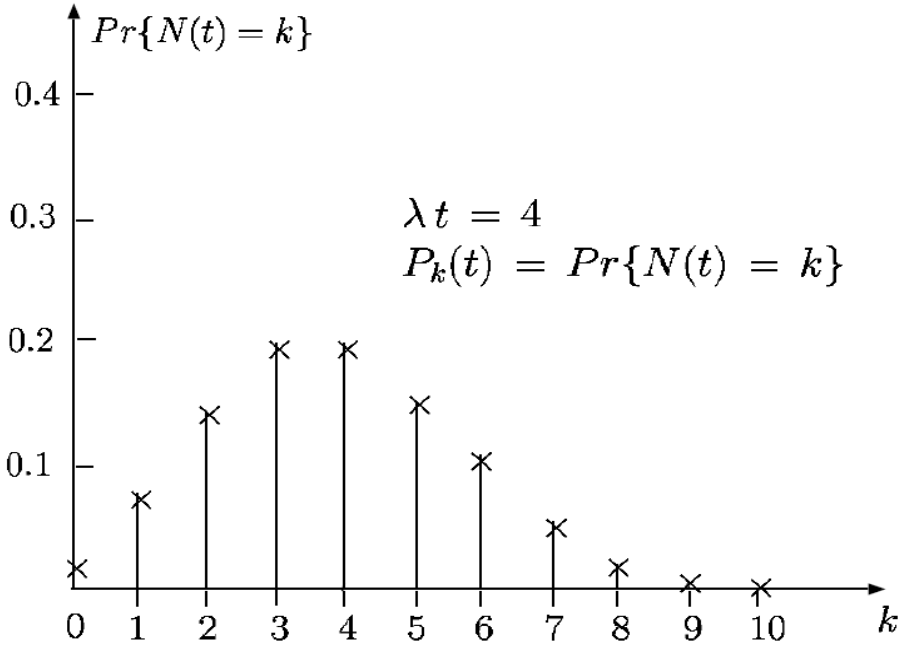
The Poisson Distribution – Example 1



k	P_k
0	0.368
1	0.368
2	0.184
3	0.061
4	0.015
5	0.003
6	$5.1 \cdot 10^{-4}$
...	...
10	$1.01 \cdot 10^{-7}$

The Poisson Distribution – Example 2



k	P_k
0	0.0183
1	0.073
2	0.146
3	0.195
4	0.195
5	0.156
6	0.104
7	0.059
8	0.029
9	0.013
10	0.005

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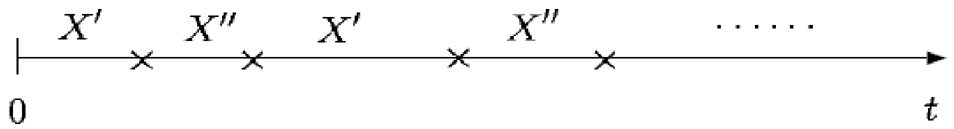
The Alternating Poisson Process

The process is constituted by a sequence of *Type I* variables X' with density $f_1(x)$ followed by a *Type II* variables X'' with density $f_2(x)$. The process starts with probability 1 with a *Type I* variable.



If we look at the sequence formed by the occurrence of the *Type II* variables, the process is an ordinary renewal process with inter-arrival time $(X' + X'')$.

The Alternating Poisson Process

Let be:

$\pi_1(t)$ - Probability *Type I* variable occurs at time t

$\pi_2(t)$ - Probability *Type II* variable occurs at time t

Type I is in use at time t if:

a) - No *Type I* event occurs in $(0 - t)$;

b) - A *Type II* event occurs in $u - u + du$ $(u < t)$, and no *Type I* events occur in $(t - u)$:

The Alternating Poisson Process

Type I variable is exponential with rate λ;
Type II variable is exponential with rate μ.

$$\pi_1(t) \;=\; \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}\, e^{-(\lambda + \mu)\, t}$$

$$\pi_2(t) \;=\; \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu}\, e^{-(\lambda + \mu)\, t}$$

$$\lim_{t \to \infty} \pi_1(t) = \frac{\mu}{\lambda + \mu} \qquad ; \qquad \lim_{t \to \infty} \pi_2(t) = \frac{\lambda}{\lambda + \mu}$$

The Alternating Poisson Process – Second Version

Homogeneous Poisson process

A *homogeneous* Poisson process is characterized by a rate parameter λ such that the number of events in time interval $[t,t + \tau]$ follows a Poison Distribution with associated parameter $\lambda\tau$. This relation is given as:

where $N(t + \tau) - N(t)$ describes the number of events in time interval $[t,t + \tau]$.
Just as a Poisson random variable is characterized by its scalar parameter λ, a homogeneous Poisson process is characterized by its rate parameter λ, which is the expected number of "events" or "arrivals" that occur per unit time.
N(t) is a sample homogeneous Poisson process, not to be confused with a density or distribution function.

$$P[(N(t+\tau)-N(t))=k]=\frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}\qquad k=0,1,\ldots$$

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The Alternating Poisson Process – Second Version

Non-Homogeneous Poisson process

In general, the rate parameter may change over time. In this case, the generalized rate function is given as $\lambda(t)$. Now the expected number of events between time *a* and time *b* is

Thus, the number of arrivals in the time interval (a, b], given as N(b)-N(a), follows a Poisson Distribution with associated parameter $\lambda_{a,b}$-

A homogeneous Poisson process may be viewed as a special case when $\lambda(t) = \lambda$, a constant rate.

$$\lambda_{a,b} = \int_a^b \lambda(t)\,dt.$$

$$P[(N(b) - N(a)) = k] = \frac{e^{-\lambda_{a,b}}(\lambda_{a,b})^k}{k!} \qquad k = 0, 1, \ldots$$

Bubble Signal Analysis - Transform Methods

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Power Spectral Density Function

$$\text{Fourier Transform} = \text{Magnitude and Phase Spectrum} = F(s) = \int_{-\infty}^{\infty} f(x)\cdot e^{-i\cdot 2\cdot\pi\cdot s}\,dx$$

$$\text{Inverse Fourier Transform} = \text{Real or Complex Time Series} = f(x) = \int_{-\infty}^{\infty} F(s)\cdot e^{-i\cdot 2\cdot\pi\cdot\omega}\,ds$$

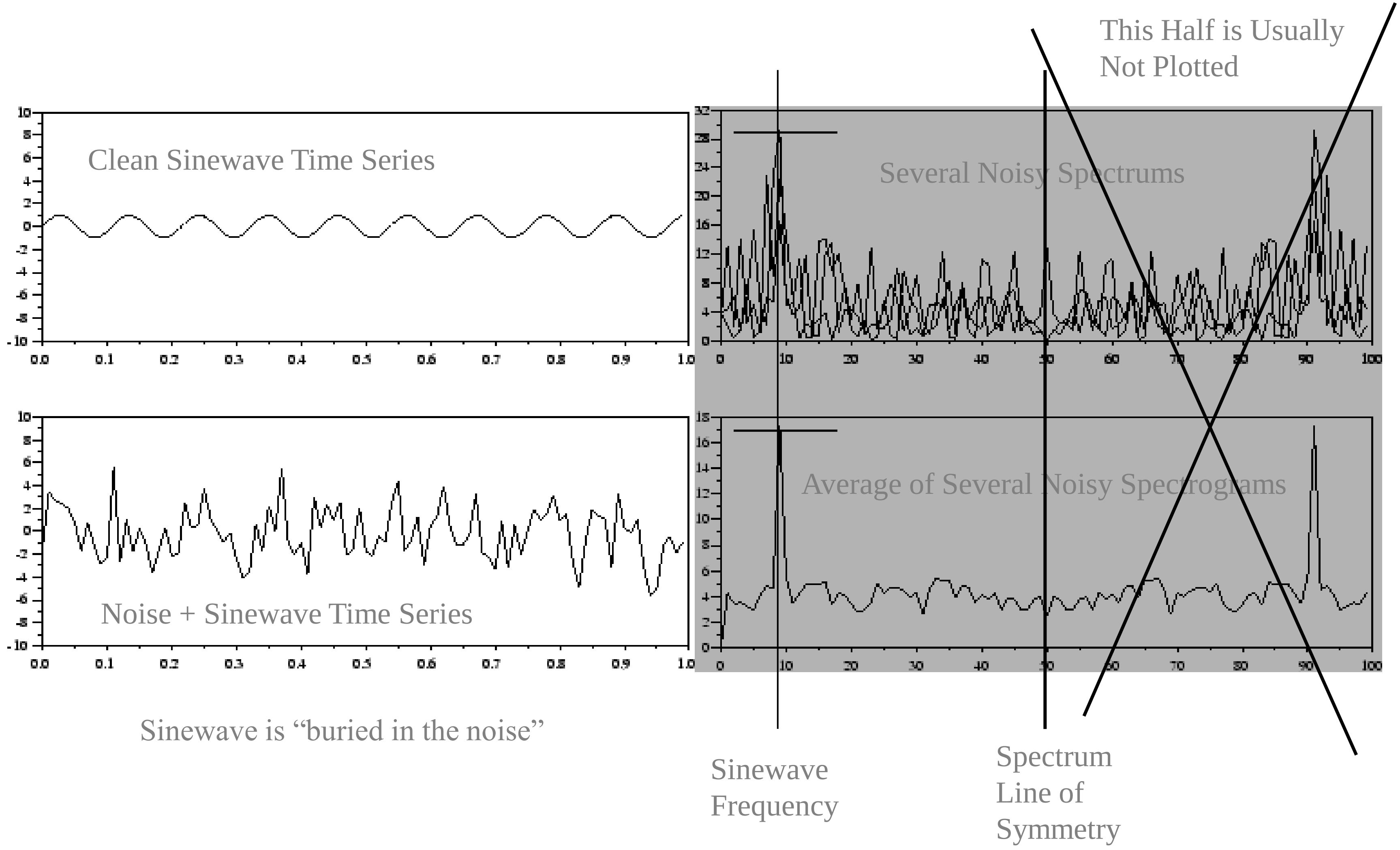
$$\text{Power Spectral Density} = \text{PSD} = \int_{-\infty}^{\infty} \left(\left|F(s)\right|\right)^2 ds \quad \text{or} \quad \int_{-\infty}^{\infty} F(s)\cdot F(s)^{\phi}\,ds$$

where $F(s)^{\phi}$ is the complex conjugate of F(s) and s is the complex frequency (j* ω)

f(x) is the time series to be analyzed and F(s) is the complex (mag and phase) Fourier Transform of the time series

The Power Spectral Density Function tells us at which frequencies there is energy within the time series that we are analyzing. A plot of amplitude, power or energy vs. frequency is called a "Spectrogram"

The PSD Function for a Noised Sine Wave



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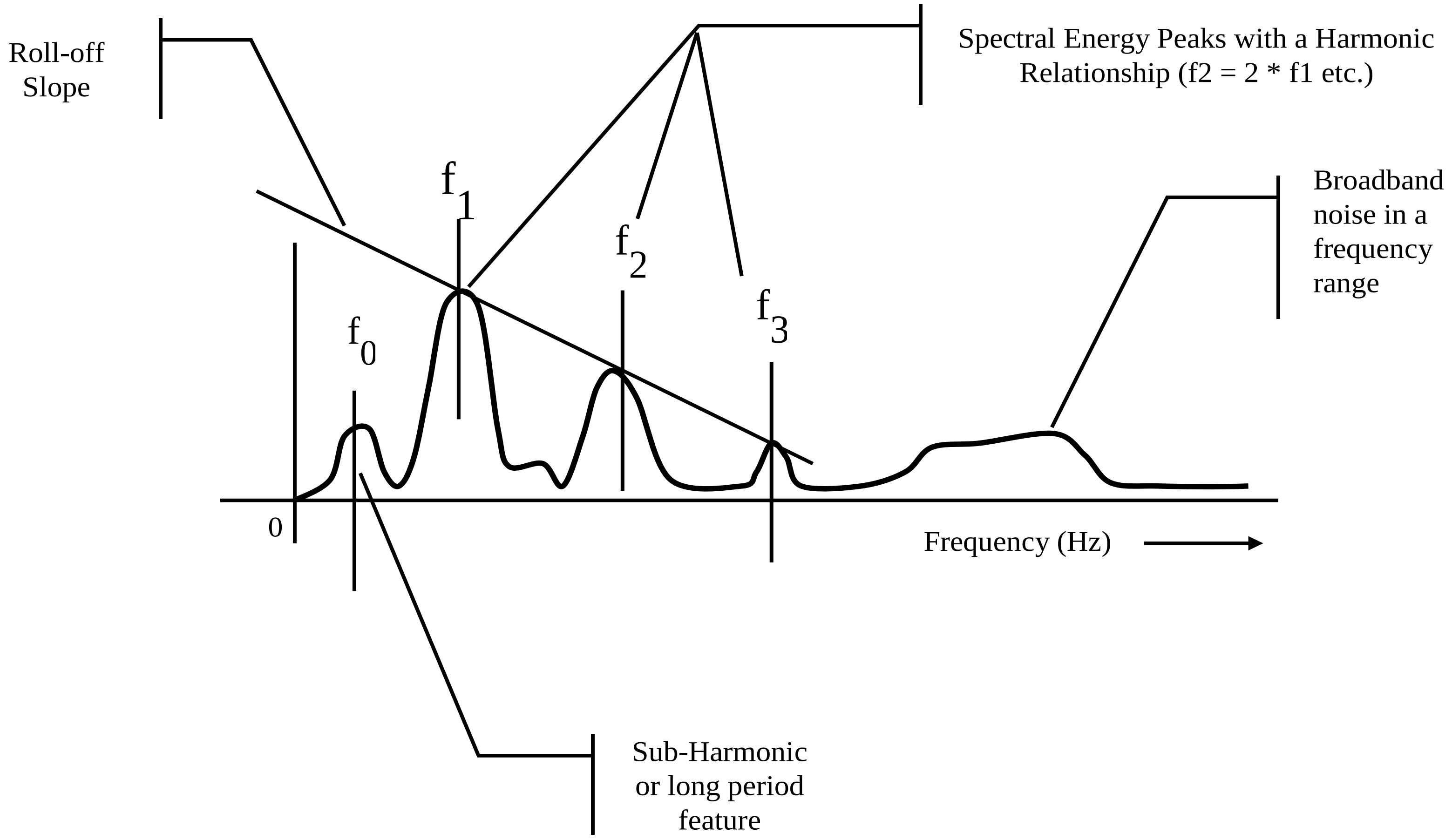
Power Spectral Density Function - continued

What to Look for When Using the Power Spectral Density Function



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The Autocorrelation Function

$$\text{Auto-Correlation Function} = \text{ACF}(\tau) = \int_{-\infty}^{\infty} F(\tau + x)\cdot F(\tau)^{\phi}\, d\tau \quad \text{or} \quad \int_{-\infty}^{\infty} \text{PSD}\cdot e^{-i\cdot 2\cdot \pi\cdot \omega}\, ds$$

where $F(\tau)^{\phi}$ is the complex conjugate of $F(\tau)$, τ is the relative correlation time delay and s is the complex frequency ($j^* \omega$)

The Autocorrelation Function measures how similar a time series is to itself when compared at different relative time delays. Because the Autocorrelation Function is the inverse Fourier transform of the Power Spectral Density Function, it represents the same information … but … in a different way.

The PSD relates the time series and its energy at different frequencies. The ACF relates the time series to a time delayed copy of itself. Because each is the Fourier transform of the other, a feature in the time series that repeats itself at a fairly regular time intervals will be represented by a peak in the Autocorrelation function at a time delay equal to the repetition interval. The same feature will appear in the Power Spectral Density plot as a “peak” at a frequency equal to the inverse of the time delay (freq = 1 / time).

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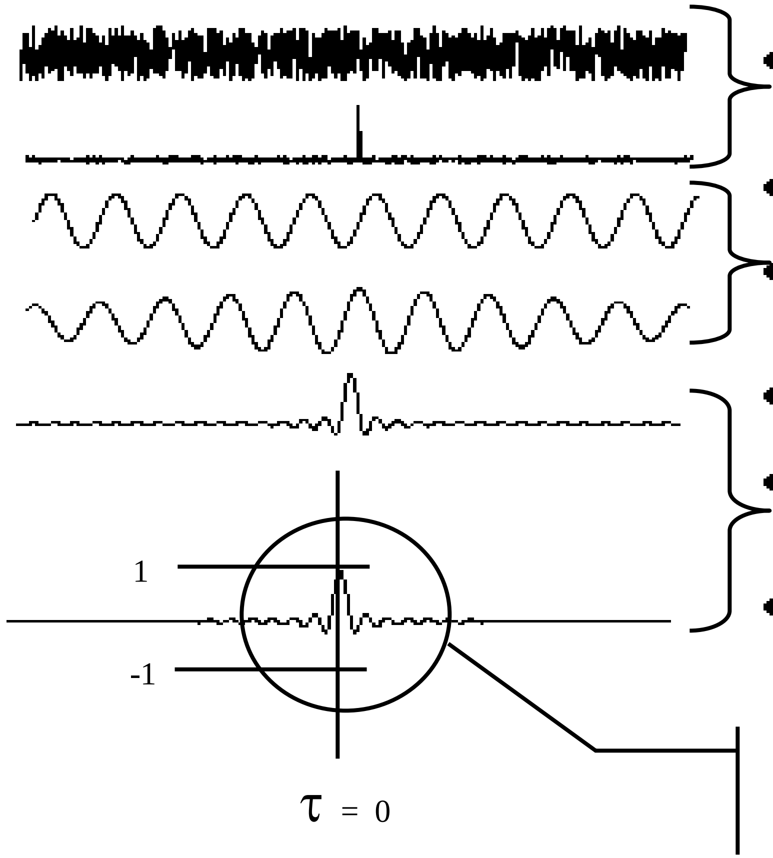
The Autocorrelation Function



- Random noise is similar to itself, and in phase, only with no time shift at all
- so its correlation function is a spike
- periodic signals go in and out of phase as they are time shifted
- so their correlation functions are periodic
- signals that last only a short while are only similar while they last
- so their correlation functions are short

Magnified and explained on the next page

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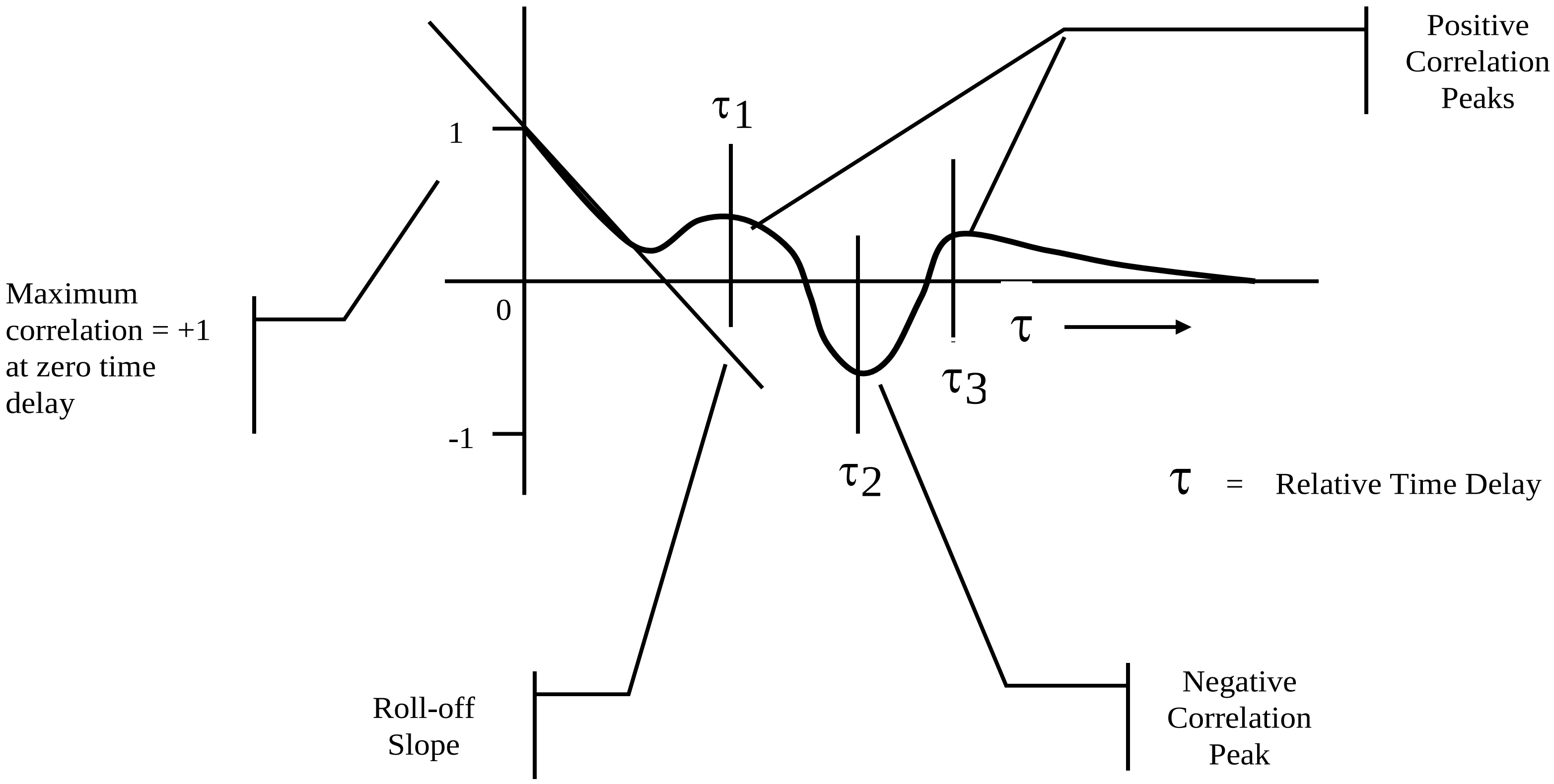
The Autocorrelation Function - Continued

What to Look for When Using the Autocorrelation Function



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Summary and Conclusions

A preliminary stochastic model is presented for the bubble generation and detection processes

Several means of processing bubble signals are presented

By these means, estimates of gas fraction may be obtained

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