

Ion Transport in Electrolyte Solutions

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Charge Mobility – Absolute – any Kind of Force

From Bockris and Reddy Vol 1 pages 370 - 371

Absolute Mobility = u_a = drift velocity / force

Let z = # charges per particle

e = charge of an electron = 4.8×10^{-10} {statcoulomb}

X = electric field {volts/cm}

$1/300$ = electrostatic unit conversion factor

Force = $F = z \cdot e \cdot X \cdot (1/300)$ {dynes}

1 dyne = 10^{-5} Newton

Force = $F = z \cdot e \cdot X \cdot (1/300) \cdot (10^{-5})$ {Newtons}

Example: $X = .05$ volt/cm

$V_{drift} = 2 \cdot 10^{-5}$ cm/sec

Force = $8 \cdot 10^{-14}$ {dynes}

Force = $8 \cdot 10^{-19}$ {Newtons}

$U_{abs} = 2.5 \cdot 10^8$ { cm / (sec * dyne) }

$U_{abs} = 2.5 \cdot 10^{13}$ { cm / (sec * Newtons) }

Charge Mobility – Conventional – Electric Force Only

From Bockris and Reddy Vol 1 pages 370 -371

$$\text{Conventional Mobility} = U_c = V_d / X = (1/300) * z * e * U_{\text{abs}} \quad \{ \text{cm}^2 / (\text{volt} * \text{sec}) \}$$

$$\text{Absolute Mobility} = U_{\text{abs}} = V_d / F = (300 * U_c) / (z * e) \quad \{ \text{cm} / (\text{sec} * \text{dyne}) \}$$

Example:

$$X = .05 \text{ volt/cm} \quad V_{\text{drift}} = 2 * 10^{-5} \text{ cm/sec}$$

$$U_{\text{conv}} = 4 * 10^{-4} \{ \text{cm}^2 / (\text{volt} * \text{sec}) \}$$

Ion Conductivity – Electric Force Only

From Bockris and Reddy Vol 1 pages 373

$$\text{Particle Flux Density} = J = \text{concentration} * \text{velocity} \quad \{ \text{moles}/(\text{cm}^2 * \text{sec}) \}$$

$$\text{Current Density} = I = z * F * \text{Conc} * \text{Vel} \quad \{ \text{amperes} / \text{cm}^2 \}$$

$$F = 9.48 * 10^4 \quad \{ \text{coulomb} / \text{mole} \} \quad \text{Conc} = \{ \text{moles} / \text{cm}^2 \}$$

$$\text{Note : } \text{Conc} = \{ \text{moles} / \text{cm}^2 \} = (1/1000) * \text{Conc} \{ \text{moles/liter} \}$$

$$\text{Conductivity} = \sigma = I / X \quad \{ \text{siemens} / \text{cm} \}$$

$$\sigma = z * F * \text{Conc} * U_{\text{conv}}$$

Absolute Mobility and Diffusion Coefficient – Einstein Relation

From Bockris and Reddy Vol 1 pages 374

$$\text{Particle Flux Density} = J_d = \text{concentration} * \text{velocity} = -D * dC/dx \quad \{ \text{moles}/(\text{cm}^2 * \text{sec}) \}$$

$$V_{\text{drift}} = U_{\text{abs}} * \text{Force} \quad \{ \text{cm}/\text{sec} \}$$

$$\text{Current Density} = I = z * F * \text{Conc} * V_d \quad \{ \text{amps} / \text{cm}^2 \}$$

$$\text{Conduction Flux Density} = J_c = I / (z * F) = \text{Conc} * V_d = \{ \text{moles} / (\text{cm}^2 * \text{sec}) \}$$

$$J_c = \text{Conc} * U_{\text{abs}} * \text{Force}$$

$$\text{Particle Flux Density Total} = J_{\text{tot}} = J_{\text{diff}} + J_{\text{cond}}$$

$$D = \text{Diff Coef} = U_{\text{abs}} * K * T \quad \{ \text{cm}^2 / \text{sec} \}$$

$$D = \text{Diff Coef} = (300 * U_c) / (z * e) * K * T \quad \{ \text{cm}^2 / \text{sec} \}$$

$$K * T = .0257 \text{ eV} = 4.12 * 10^{-21} \quad \{ \text{Joules} \} = \\ RT/F = .0257 \text{ eV}$$

$$dC/dx = (C * U_{\text{abs}} * F) / D$$

Joules, Ergs and Electron Volts

$$1 \text{ joule} = 1 * 10^7 \text{ ergs}$$

$$1 \text{ electron volt} = 1 \text{ eV} = 1.6 * 10^{-19} \text{ joule} = 1.6 * 10^{-12} \text{ ergs}$$

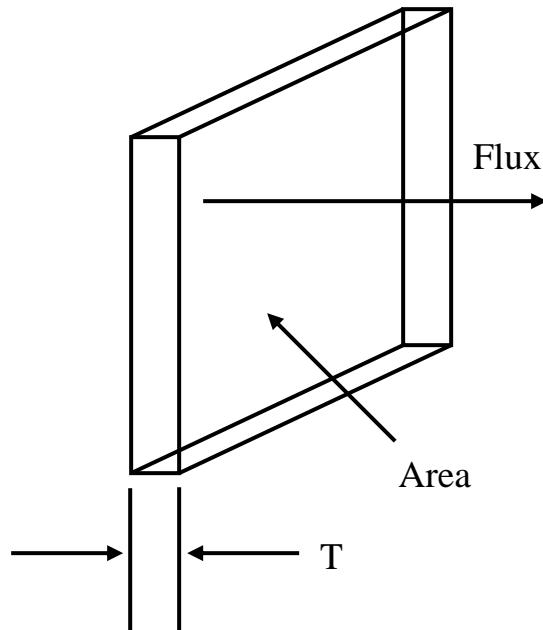
$$\text{Boltzman's constant} = K_b = 1.3807 * 10^{-23} \text{ joule / K} = 1.3807 * 10^{-16} \text{ erg per k}$$

$$T_0 = 273 \text{ K} \quad T_{27} = 300$$

$$K * T = 1.3807 * 10^{-16} * 300 = 3.9e-14 \text{ ergs}$$

$$1 \text{ erg} = 1 \text{ dyne} * \text{cm}$$

Example - Ionic Conduction in 4 Molar H₂SO₄ – 1 Amp / cm²



$$V = T / \sigma = T / .75 = .267 \text{ Volts}$$

$$X = \text{electric field} = .267 / .2 = 1.34 \text{ volts / cm}$$

$$V_{\text{drift_proton}} = U_c * X = .003 * 1.34 = .004 \text{ cm/sec}$$

$$= .04 \text{ mm/sec} = 40 \text{ microns/sec}$$

or 100 seconds to cross the 2 mm channel

$$D = U_{\text{abs}} * K * T = U_{\text{abs}} * (R * T / F) \{ \text{cm}^2 / \text{sec} \}$$

$$T = .2 \text{ cm} \quad A = 1 \text{ cm}^2 \quad I = 1 \text{ amp/cm}^2$$

$$\text{Conc} = 4 \text{ moles/liter} = .004 \text{ moles / cm}^3$$

$$\text{Specific Conductivity} = \sigma = .75 \text{ Siemens / cm}$$

$$U_{\text{proton_conv}} = 3 * 10^{-3} \text{ cm}^2 / (\text{sec} * \text{volt})$$

$$U_{\text{sulfate_conv}} = .5 * 10^{-3} \text{ cm}^2 / (\text{sec} * \text{volt})$$

Bockris & Reddy p471

$$U_{\text{proton_abs}} = (300 / z * e) * U_c$$

$$= (300 / 4.8 * 10^{-10}) * .003$$

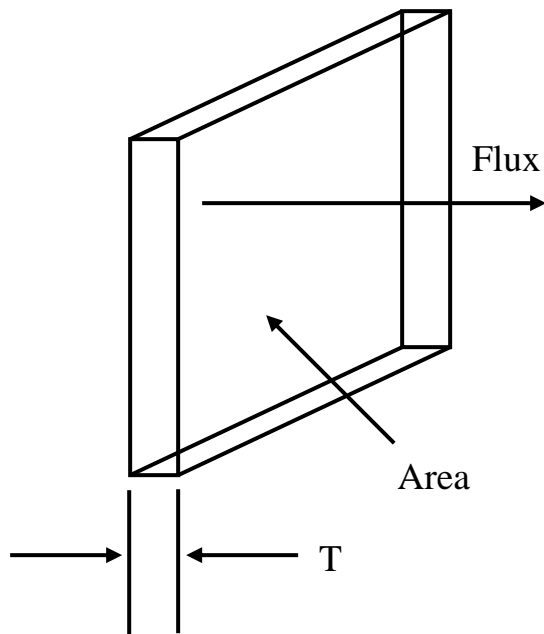
$$= 1.875 * 10^9 \text{ cm}/(\text{sec} * \text{dyne})$$

$$D_{\text{proton}} = U_{\text{abs}} * K * T$$

$$= 1.875 * 10^9 * 3.9e-14$$

$$= 7.3 * 10^{-5} \text{ cm}^2/\text{sec}$$

Example - Diffusion Current Density of 1 Amp/cm²



$$D_{\text{proton}} = 7.3 \times 10^{-5} \text{ \{ cm}^2 / \text{sec} \} \text{ Bockris - p 471}$$

$$T = .2 \text{ cm} \quad A = 1 \text{ cm}^2 \quad I = 1 \text{ amp/cm}^2$$

$$\text{Specific Conductivity} = \sigma = .75 \text{ Siemens / cm}$$

$$\text{Electric Field} = E = dV/dx = 1.34 \text{ Volts/cm (for 1 amp/cm}^2)$$

Assume 2 molar MeOH at the anode inlet is half reacted then the concentration of newly created protons at the cathode facing surface of the anode must be $1 \times 6 \text{ m/liter} = .006 \text{ mole/cm}^3$
 In the steady state all of this must be reacted in the surface region

$$\text{Current Density} = i = -F^2 \cdot dE/dx \cdot z^2 \cdot U_{\text{conv}} \cdot C$$