TEM_{00} Gauss-beam described with ray-optics.

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Example of an optical system consisting of two lenses with focal lengths f_1 and f_2 at positions z_1 and z_2 respectively. These lenses are the uncorrected focal beam of a laser-optic system.

all dimensions are mm

$z_1 := 10$ $f_1 := 20$ $z_2 := 15$ $f_2 := -10000$ the minus means meniscus lens

The calculation starts with a Gaussian beam with wavelength λ and a waist w_0 at z_0:

$\lambda := .001$ $w_0 := 5$ $z_0 := 0$ wavelength is 1 micron

The system can be described using ABCD matrices for the propagation (M_0, M_1, and M_2) and for the two lenses (M_{L1} and M_{L2}):

$$B_0(z) := \text{if}\left[(z \ge z_0)\cdot(z \le z_1), z - z_0, z_1 - z_0\right]$$

$$M_0(z) := \begin{pmatrix} 1 & B_0(z) \\ 0 & 1 \end{pmatrix}$$

$$B_1(z) := \text{if}\left[z \le z_1, 0, \text{if}\left[(z \le z_2), z - z_1, z_2 - z_1\right]\right]$$

$$M_1(z) := \begin{pmatrix} 1 & B_1(z) \\ 0 & 1 \end{pmatrix}$$

$$B_2(z) := \text{if}(z \ge z_2, z - z_2, 0)$$

$$M_2(z) := \begin{pmatrix} 1 & B_2(z) \\ 0 & 1 \end{pmatrix}$$

$$M_{L1} := \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} \qquad M_{L2} := \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}$$

The beam is calculated from z_0 to z_{max}, using N rays: $z_{max} := 101$ $N := 20$

$z := z_0 .. z_{max}$ $i := 0 .. N$

At the waist at z_0 the Gaussian beam can be described with rays of which the divergence and positions lying on an upright ellipse (χ=0):

$$\theta_{max} := \frac{\lambda}{\pi \cdot w_0} \qquad \chi := 0 \qquad x(\phi) := w_0 \cdot \sin(\phi + \chi) \qquad \theta(\phi) := \theta_{max} \cdot \cos(\phi)$$

$\phi := 0, 0.1\cdot\pi .. 2\cdot\pi$

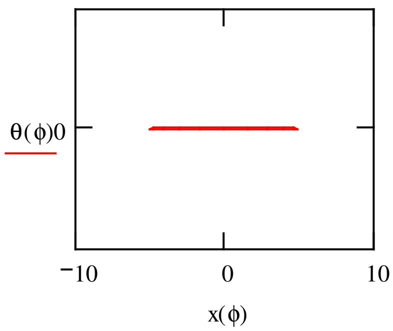

Fig. 1. The positions and divergences lie on an ellipse that is upright in the beam waist.

Definition of the positions and divergences of the N rays at z_0:

$$\Theta_i := \frac{2 \cdot i}{N} \cdot \pi \qquad \theta_{1_i} := \theta(\Theta_i) \qquad x_{1_i} := x(\Theta_i)$$

Or in vector notation:

$$R_1^{\langle i \rangle} := \begin{pmatrix} x_{1_i} \\ \theta_{1_i} \end{pmatrix}$$

After propagation of the beam from z_0 to z the position and divergency follow from:

$$R(z) := M_2(z) \cdot M_{L2} \cdot M_1(z) \cdot M_{L1} \cdot M_0(z) \cdot R_1$$

The positions and divergences of the N rays at z are:

$$x(z) := \left(R(z)^T\right)^{\langle 0 \rangle} \qquad \theta(z) := \left(R(z)^T\right)^{\langle 1 \rangle}$$

$$i_z := 0 .. z_{max} - z_0$$

$$Z_{i_z} := z_0 + i_z \qquad X_{i, i_z} := x\left(Z_{i_z}\right)_i$$

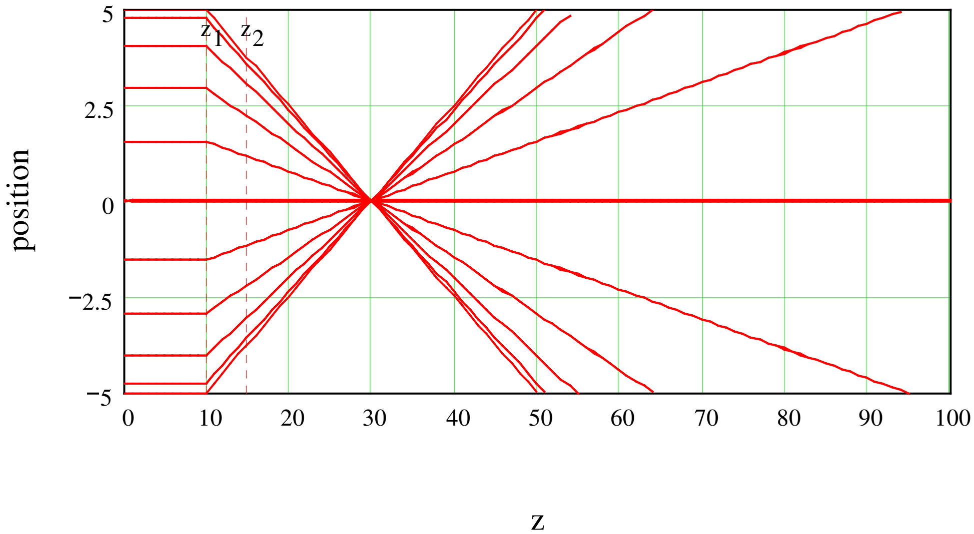

Fig. 2a. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. Expanded vertical axis. This is the ray trace for the unmodified fiber optic laser head.

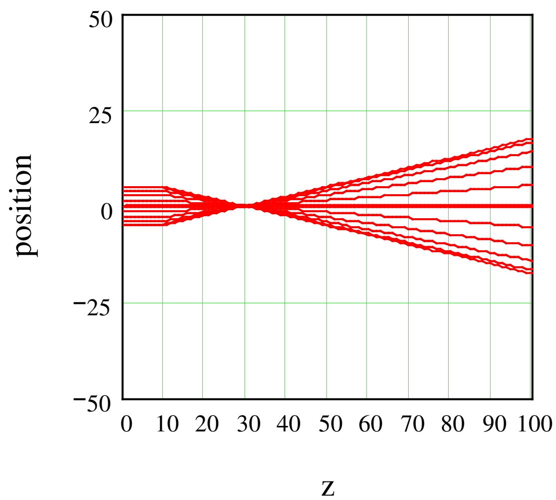

Fig. 2b. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. True Scale This is the ray trace for the unmodified fiber optic laser head.

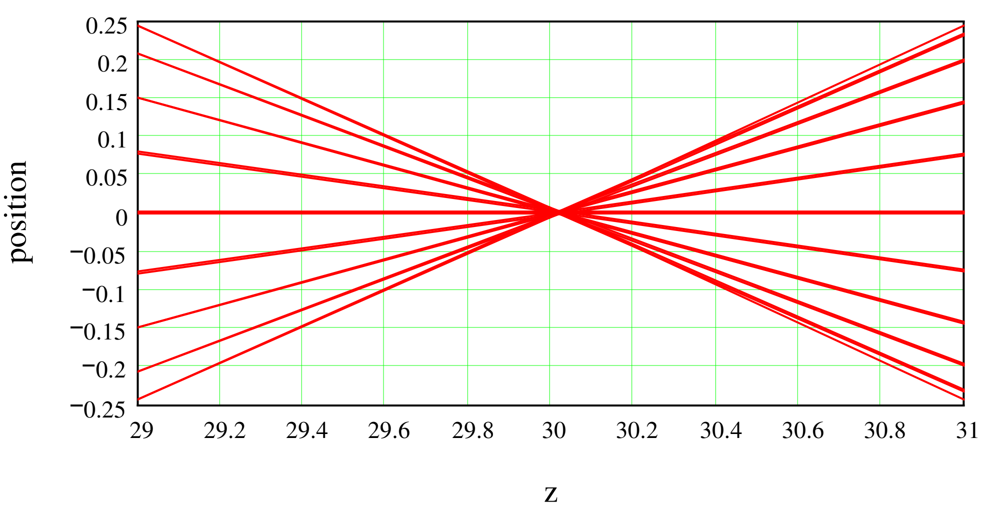

Fig. 2c. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. Highly expanded focal zone. This is the ray trace for the unmodified fiber optic laser head.

Now we do the whole thing all over again. This time we insert a negative (meniscus) lens into the optic path in order to extend the focal distance out to a more useful length. We see that a focal length of -20 mm provides a useful solution.

Example of an optical system consisting of two lenses with focal lengths f_1 and f_2 at positions z_1 and z_2 respectively:

all dimensions are mm

$z_1 := 5 \quad f_1 := 20 \qquad z_2 := 10 \quad f_2 := -20$ the minus means meniscus lens

The calculation starts with a Gaussian beam with wavelength λ and a waist w_0 at z_0:

$\lambda := .001 \quad w_0 := 5 \qquad z_0 := 0$ wavelength is 1 micron

The system can be described using ABCD matrices for the propagation (M_0, M_1, and M_2) and for the two lenses (M_{L1} and M_{L2}):

$$B_0(z) := \text{if}\left[(z \ge z_0)\cdot(z \le z_1), z - z_0, z_1 - z_0\right]$$

$$M_0(z) := \begin{pmatrix} 1 & B_0(z) \\ 0 & 1 \end{pmatrix}$$

$$B_1(z) := \text{if}\left[z \le z_1, 0, \text{if}\left[(z \le z_2), z - z_1, z_2 - z_1\right]\right]$$

$$M_1(z) := \begin{pmatrix} 1 & B_1(z) \\ 0 & 1 \end{pmatrix}$$

$$B_2(z) := \text{if}(z \ge z_2, z - z_2, 0)$$

$$M_2(z) := \begin{pmatrix} 1 & B_2(z) \\ 0 & 1 \end{pmatrix}$$

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The beam is calculated from z_0 to z_{max}, using N rays: $z_{max} := 101 \quad N := 20$

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$\phi := 0, 0.1 \cdot \pi .. 2 \cdot \pi$

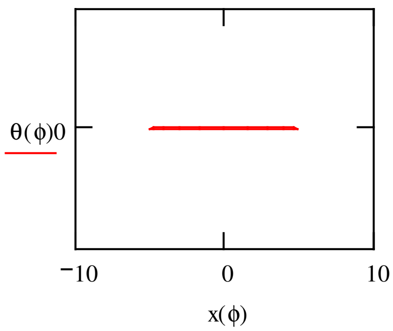

Fig. 1. The positions and divergences lie on an ellipse that is upright in the beam waist.

Definition of the positions and divergences of the N rays at z_0:

$$\Theta_i := \frac{2 \cdot i}{N} \cdot \pi \qquad \theta 1_i := \theta(\Theta_i) \qquad x1_i := x(\Theta_i)$$

Or in vector notation: $$R1^{\langle i \rangle} := \begin{pmatrix} x1_i \\ \theta 1_i \end{pmatrix}$$

After propagation of the beam from z_0 to z the position and divergency follow from:

$$R(z) := M_2(z) \cdot M_{L2} \cdot M_1(z) \cdot M_{L1} \cdot M_0(z) \cdot R_1$$

The positions and divergences of the N rays at z are:

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$$i_z := 0 .. z_{max} - z_0$$

$$Z_{i_z} := z_0 + i_z \qquad X_{i, i_z} := x\left(Z_{i_z}\right)_i$$

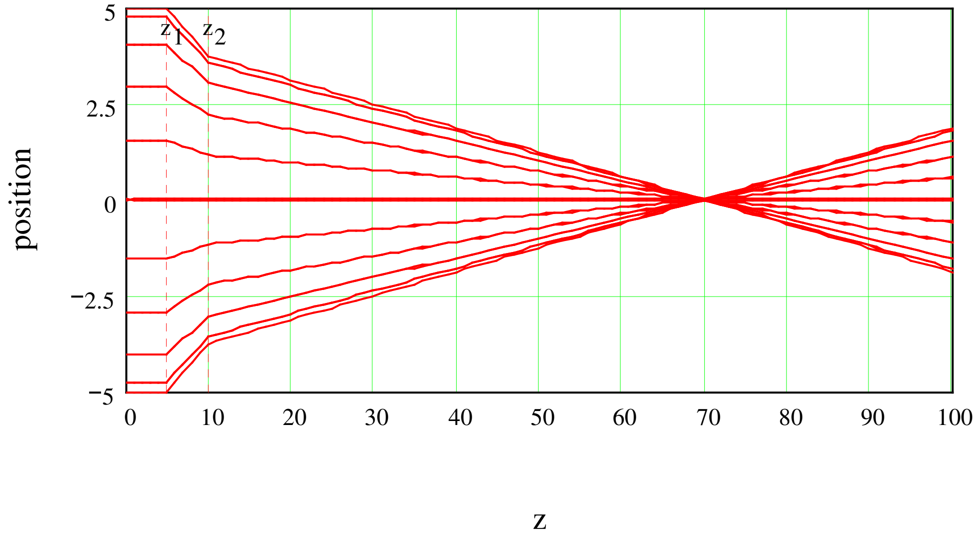


Fig. 2a. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. Expanded vertical axis. This is the ray trace for the modified fiber optic laser head.

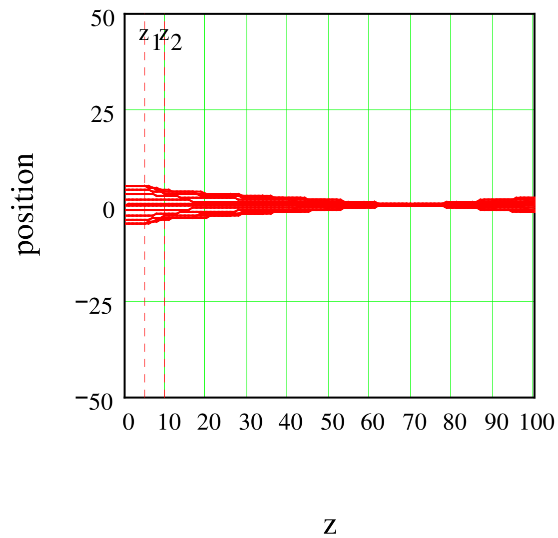


Fig. 2b. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. True Scale. This is the ray trace for the modified fiber optic laser head.

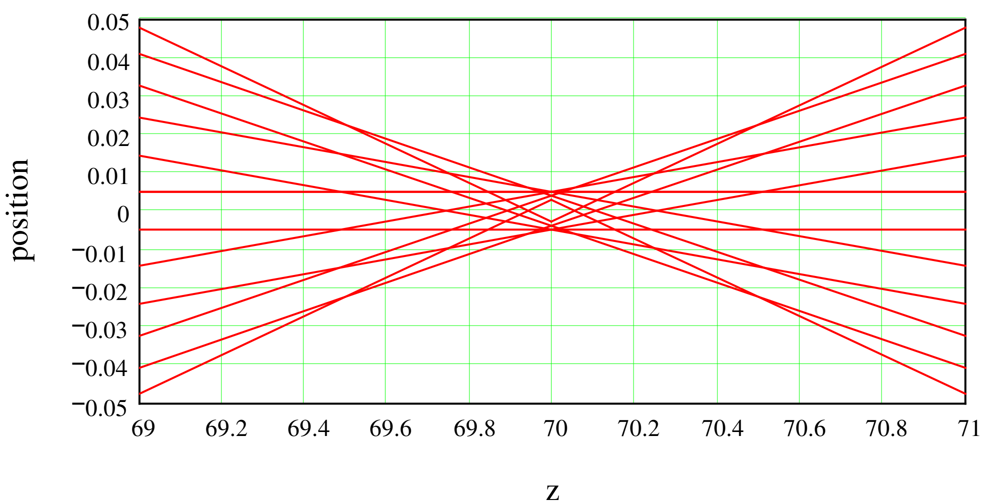
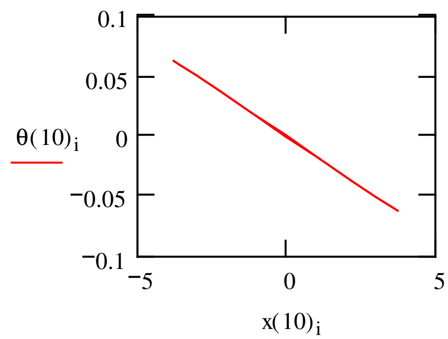
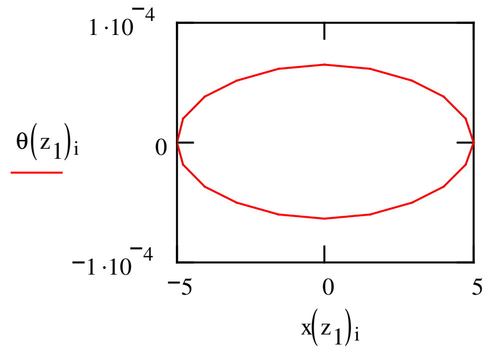


Fig. 2c. x(z) as a function of z for the N rays demonstrating the propagation of a Gaussian beam. Highly expanded focal zone. This is the ray trace for the modified fiber optic laser head. Notice that the "zone of confusion" seems to show a useful "circular ring" pattern.





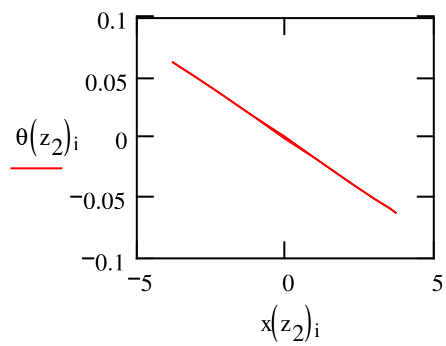


Fig. 3. The positions and divergences at several values of z.

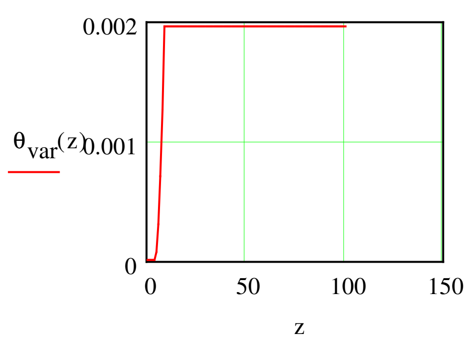
It is illustrative to calculate the beam quality which remains constant according to Liouvilles theorem and should be unity for a Gaussian beam (diffraction limited):

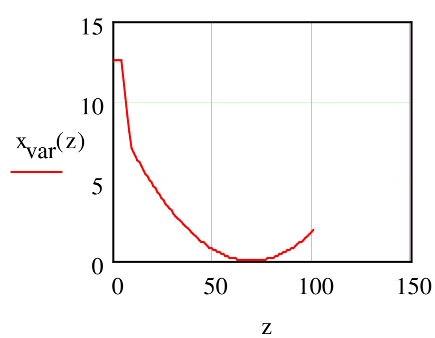
For this we need the first moment of xθ and the second moments of x and θ:

$$x\theta_{mean}(z) := \frac{1}{N}\cdot\sum_{i=0}^{N-1}\left(x(z)_i\cdot\theta(z)_i\right) \qquad x_{var}(z) := \frac{1}{N}\cdot\sum_{i=0}^{N-1}\left(x(z)_i\right)^2 \qquad \theta_{var}(z) := \frac{1}{N}\cdot\sum_{i=0}^{N-1}\left(\theta(z)_i\right)^2$$

The beam quality (or M^2 factor, or 'Times Diffraction Limited' factor) follows from:

$$\text{BeamQuality}(z) := \frac{2\cdot\pi}{\lambda}\cdot\sqrt{x_{var}(z)\cdot\theta_{var}(z) - x\theta_{mean}(z)^2}$$





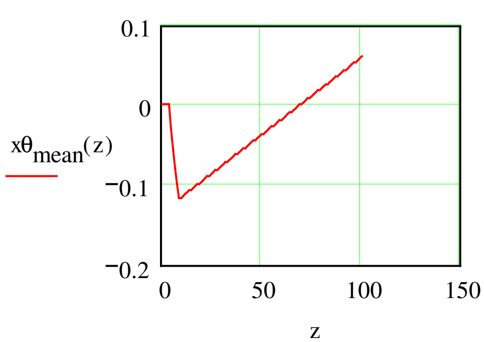


Fig. 4. The first and second moments as a function of z.

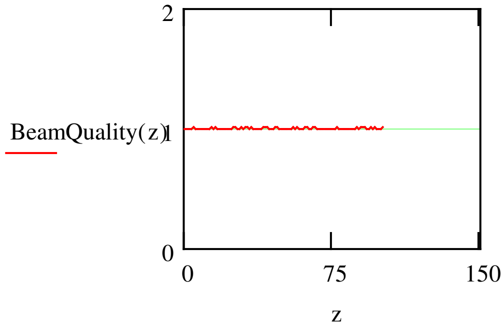
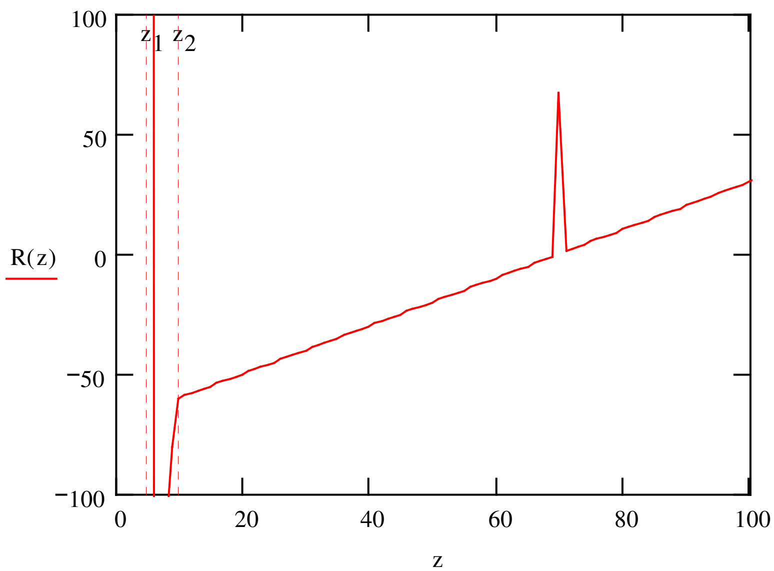


Fig. 5. The beam quality as a function of z (this should be unity for a diffraction limited Gaussian beam).

The beam radius (z), and width (envelope), w(z), can be calculated from the moments:

$$R(z) := \frac{x_{var}(z)}{x\theta_{mean}(z)} \qquad w(z) := 2\cdot\sqrt{x_{var}(z)}$$



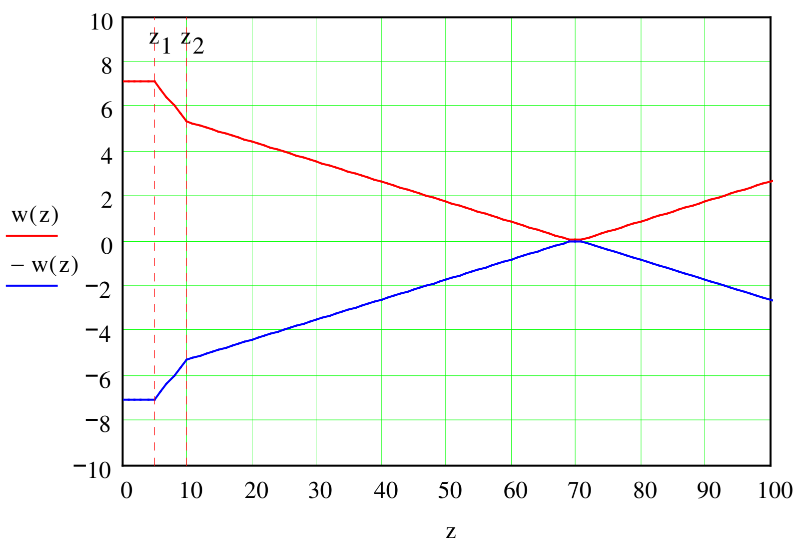


Fig. 6a. The beam radius and width calculated from the moments.

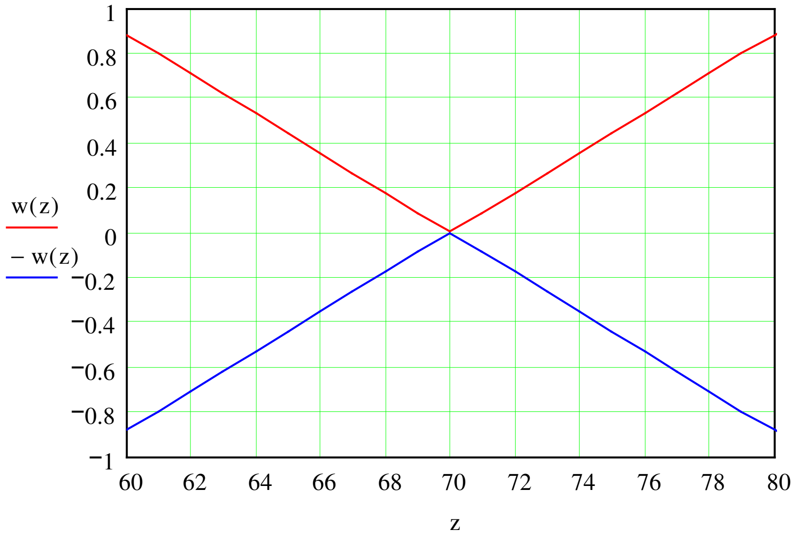


Fig. 6b. The beam radius and width calculated from the moments.

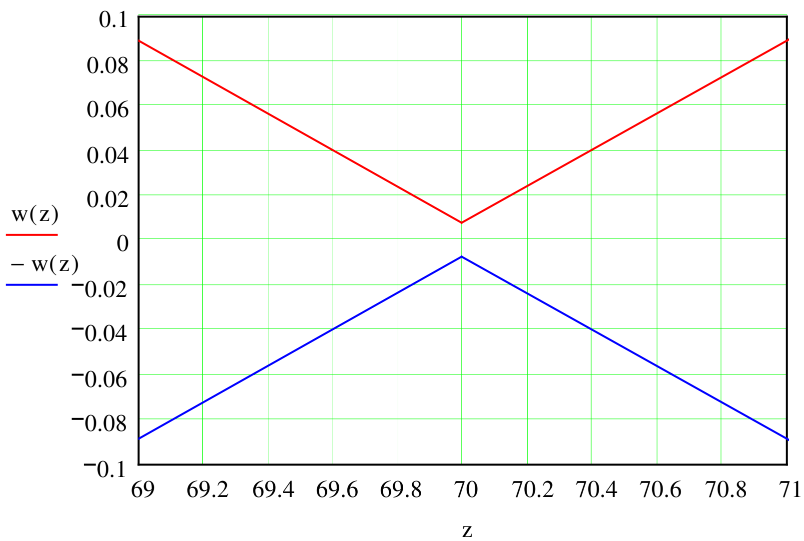


Fig. 6c. The beam radius and width calculated from the moments.

Last we calculate the diffraction limited beam waist for an ideal lens with parameters as follows (from O'Shea page 232):

convergence double angle: $\text{theta\_degrees} := 5$

$\text{theta\_rad} := \text{theta\_degrees} \cdot \frac{\pi}{180}$ $\text{theta\_rad} = 0.087$ rad $\lambda = 1 \times 10^{-3}$ mm

$\text{Dwaist} := \frac{4\lambda}{\pi \cdot \text{theta\_rad}}$ $\text{Dwaist} = 0.015$ mm

the Rayleigh range is: $\text{Range} := \frac{\text{Dwaist}}{\text{theta\_rad}}$ $\text{Range} = 0.167$ mm

The calculated beam waist and Rayleigh range are in good agreement with the previous calculations