Self Focusing

of a

Propagating Monochromatic Gaussian Beam Wave

with

Transverse Linear Polarization

A Finite Element Analysis (FEA) using flexPDE

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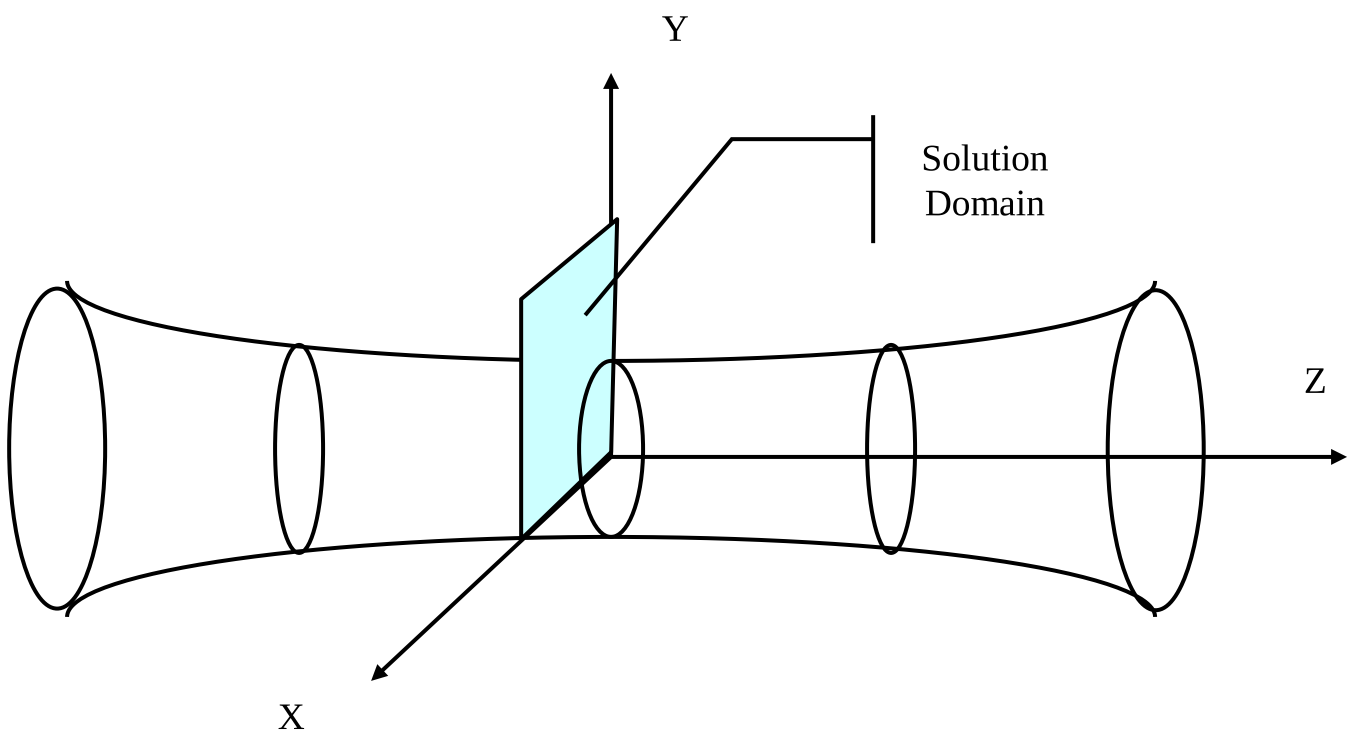
Geometry for the Finite Element Model Solution



Geometry for the Finite Element Model Solution

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Assumptions

1. Paraxial Approximation
2. Linear transverse polarization
3. Solution will be in x-y plane at z = 0

from " Electrodynamics of Continuous Media" Landau and Lifshitz 2nd ed page 380 :

$$i \cdot k_0 \cdot \left(\frac{d}{dz}E\right) + i \cdot \left(\frac{1}{u}\right) \cdot \left(\frac{d}{dt}E\right) = -\frac{1}{2} \cdot del2_{xy}(E) - \eta(\omega) \cdot (|E|)^2 \cdot E \qquad del2_{xy}(E) = \frac{d^2}{dx^2}E + \frac{d^2}{dy^2}E$$

Setting $\frac{d}{dz}(E) = 0$ and rearanging the terms :

$$del2_{xy}(E) + 2 \cdot \eta(\omega) \cdot E^2 \cdot E = -2 \cdot \left(\frac{k_0}{u}\right) \cdot \frac{d}{dt}E \cdot i$$

to simplify life ... and ... without loss of generality ... I set $2 \cdot \eta(\omega) = 1$ and $2 \cdot \left(\frac{k_0}{u}\right) = 1$

then after a certain amount of equation manipulation I find :

$$i \cdot del2_{xy}(E) + i \cdot \left(E^2 \cdot E\right) = -\frac{d}{dt}E$$

To set up for FEA solution in terms of a monochromatic E field, I now split this equation into real and imaginary parts :

Real Part :

$$del2_{xy} \cdot (E_i) + \left(E^2 \cdot E_i\right) = -\frac{d}{dt}E_r$$

Imaginary Part :

$$del2_{xy} \cdot (E_r) + E^2 \cdot E_r = \frac{d}{dt}E_i$$

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Now only one setup task remains … namely … determine $(E)^2$:

From basic complex number algebra:

$$E := \sqrt{(E_r)^2 + (E_i)^2} \qquad E^2 = (E_r)^2 + (E_i)^2$$

Now … finally … I can write the partial differential equation pair that I will numerically solve:

Real Part :

$$del2_{xy} \cdot (E_i) + \left[(E_r)^2 + (E_i)^2 \right] \cdot E_i = -\frac{d}{dt} E_r$$

Imaginary Part :

$$del2_{xy} \cdot (E_r) + \left[(E_r)^2 + (E_i)^2 \right] \cdot E_r = \frac{d}{dt} E_i$$

Finite Element Analysis (FEA) Solution Results

using flexPDE

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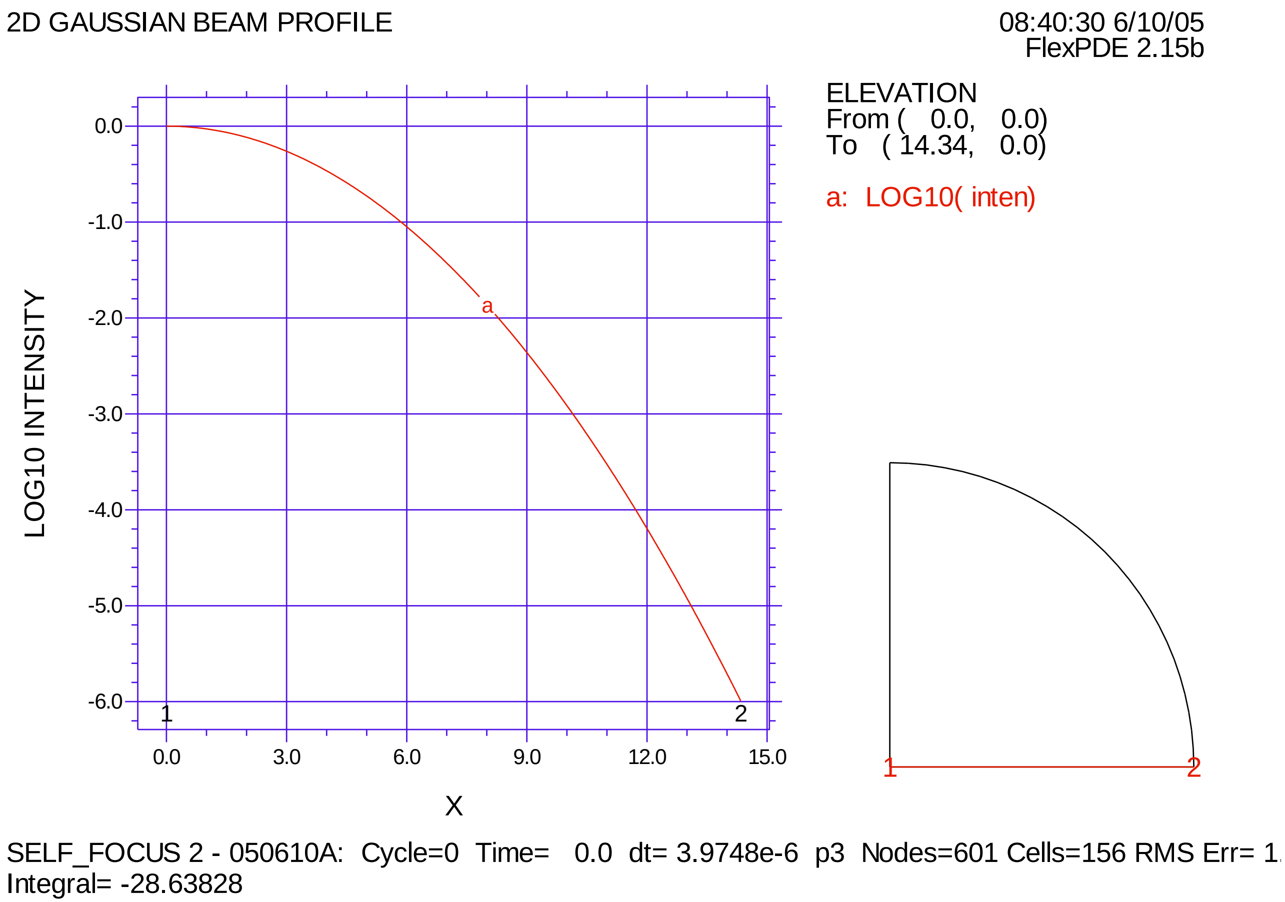
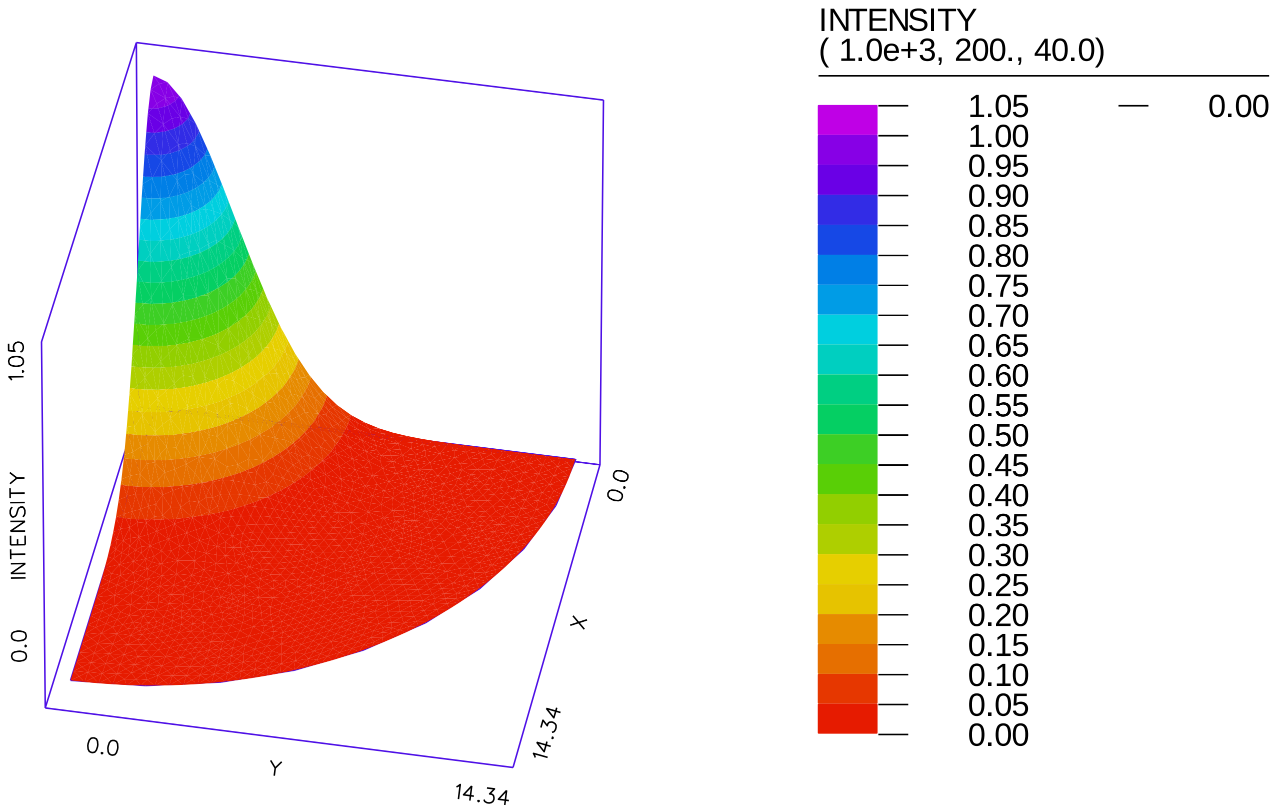


Optical Field Intensity at To - log10 Scale

2D GAUSSIAN BEAM PROFILE

08:40:30 6/10/05
FlexPDE 2.15b



SELF_FOCUS 2 - 050610A: Cycle=0 Time= 0.0 dt= 3.9748e-6 p3 Nodes=601 Cells=156 RMS Err= 1.
Integral= 11.71091

Optical Field Intensity at To - linear Scale

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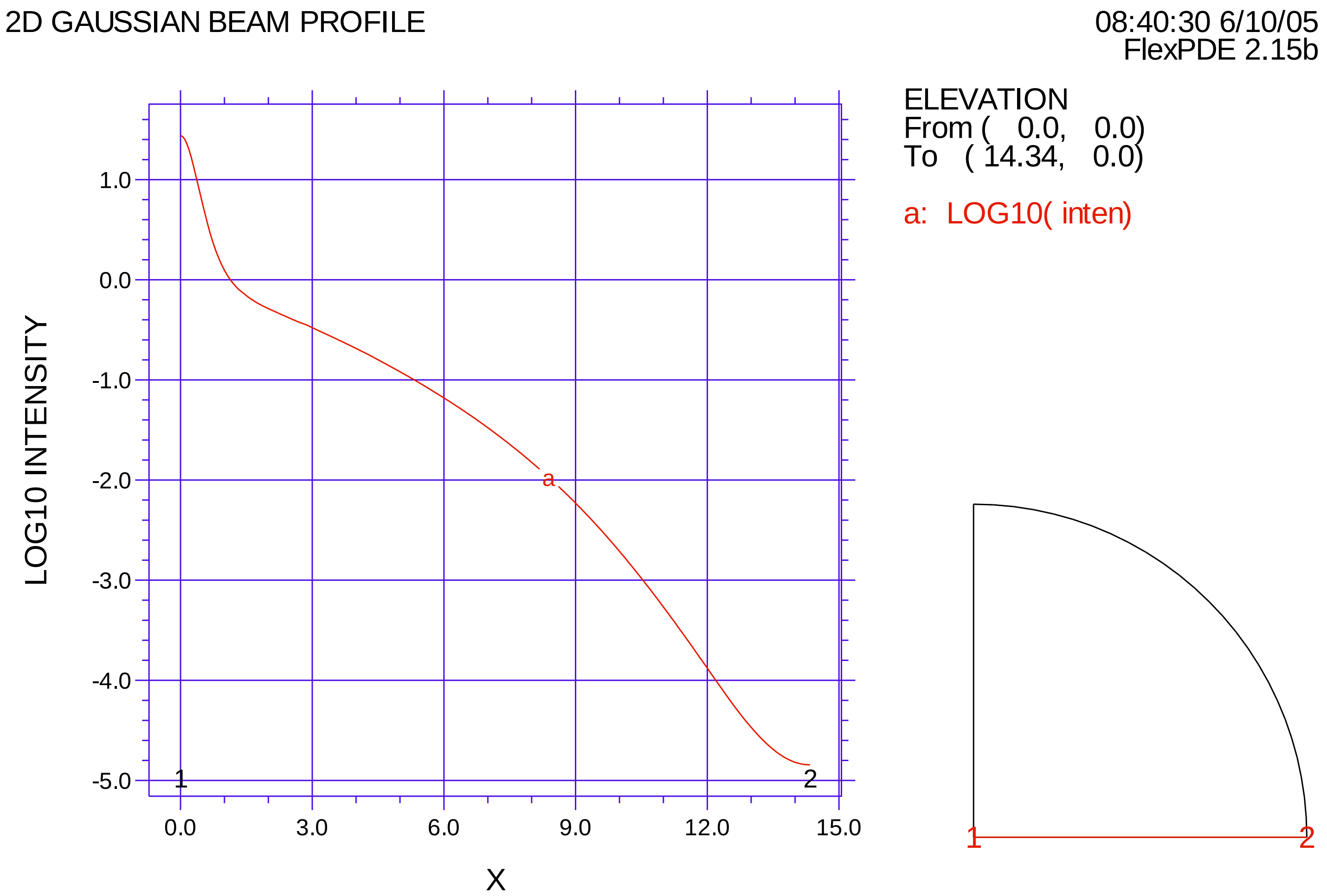

SELF_FOCUS 2 - 050610A: Cycle=103 Time= 2.25 dt= 3.9496e-3 p3 Nodes=703 Cells=183 RMS Err=
Integral= -26.68656

Optical Field Intensity at T2.25 - log10 Scale

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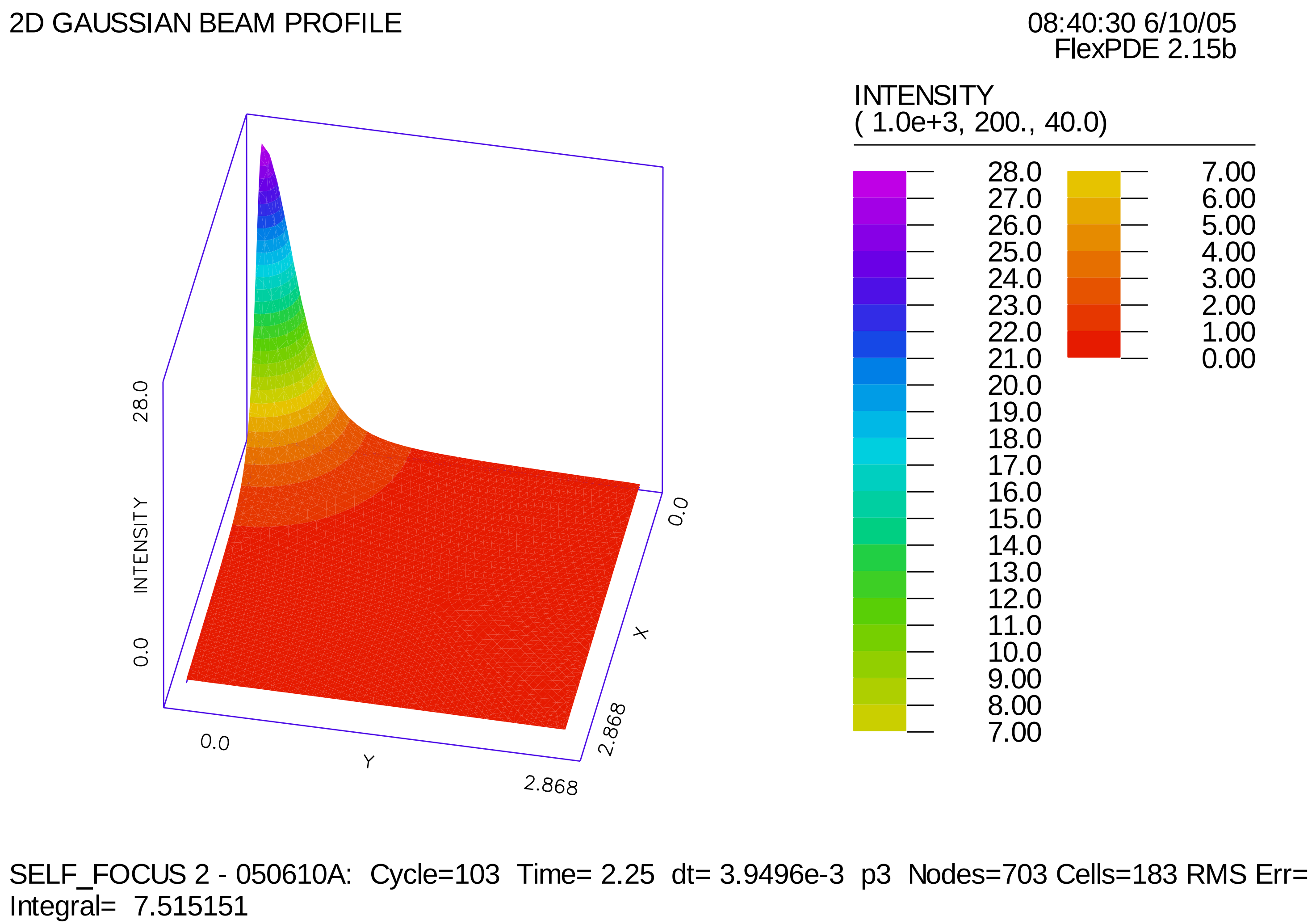

Optical Field Intensity at T2.25 - linear Scale

Summary

A simple yet useful numerical model has been made for the purpose of understanding the Nature of self focusing of a monochromatic Gaussian beam.

The model shows general beam intensity characteristics during a turn on transient

With further work, many kinds of additional information can be obtained. This includes, but is not limited to, information about the effect of nonlinearity coefficient of the medium And what happens when the beam isn't circularly symmetric