Toothbrush Bristle Induced Fluid Flow Near a Molar Tooth and Gum Line

Fluid is Water - Maximum Velocity is .42 m/sec

A Finite Element Analysis (FEA) using flexPDE

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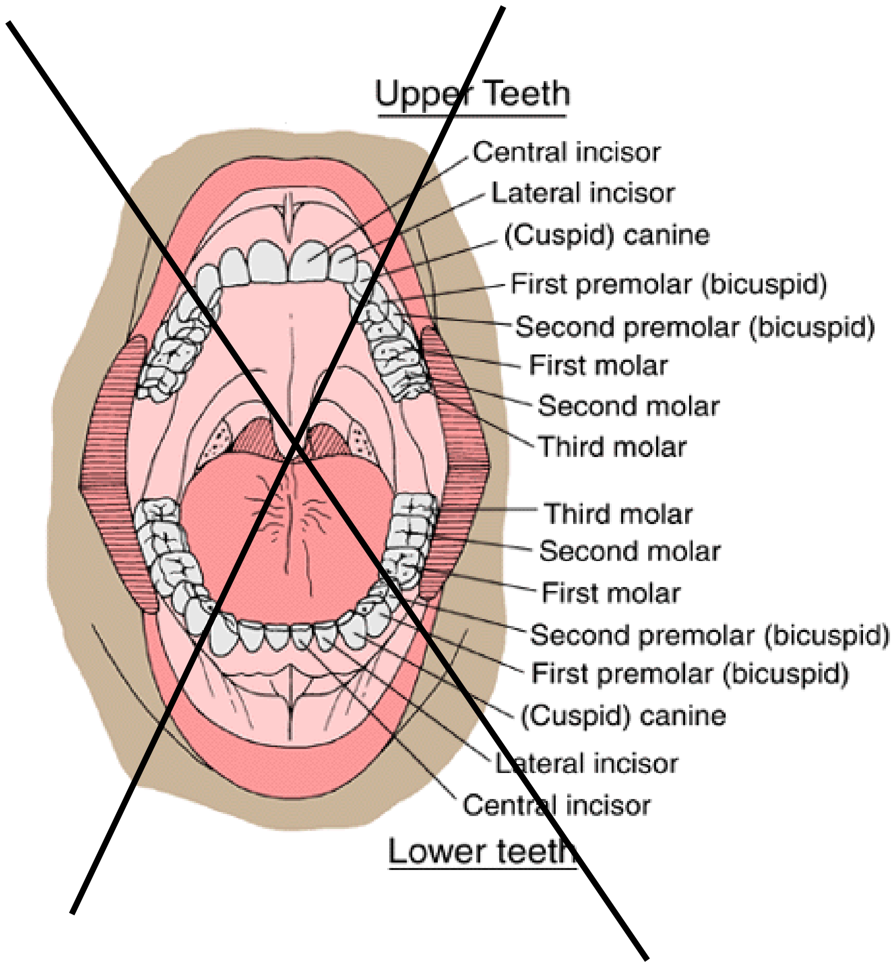
Goals for the Numerical Experiment

1. Obtain experience modeling fluid dynamic situations involving toothbrushes, teeth and gums

2. Learn where physical sensors might be placed in order to make accurate physical measurements

Numerical Experiment Setup

Full Mouth is Too Complicated to Efficiently Model



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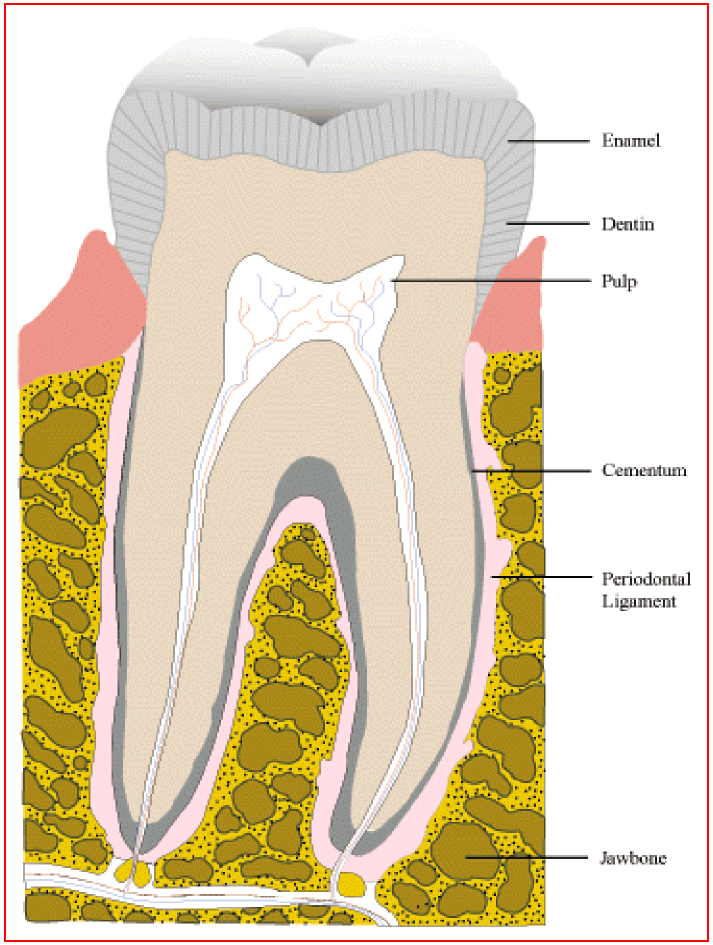
A Single Tooth and Gumline Are OK



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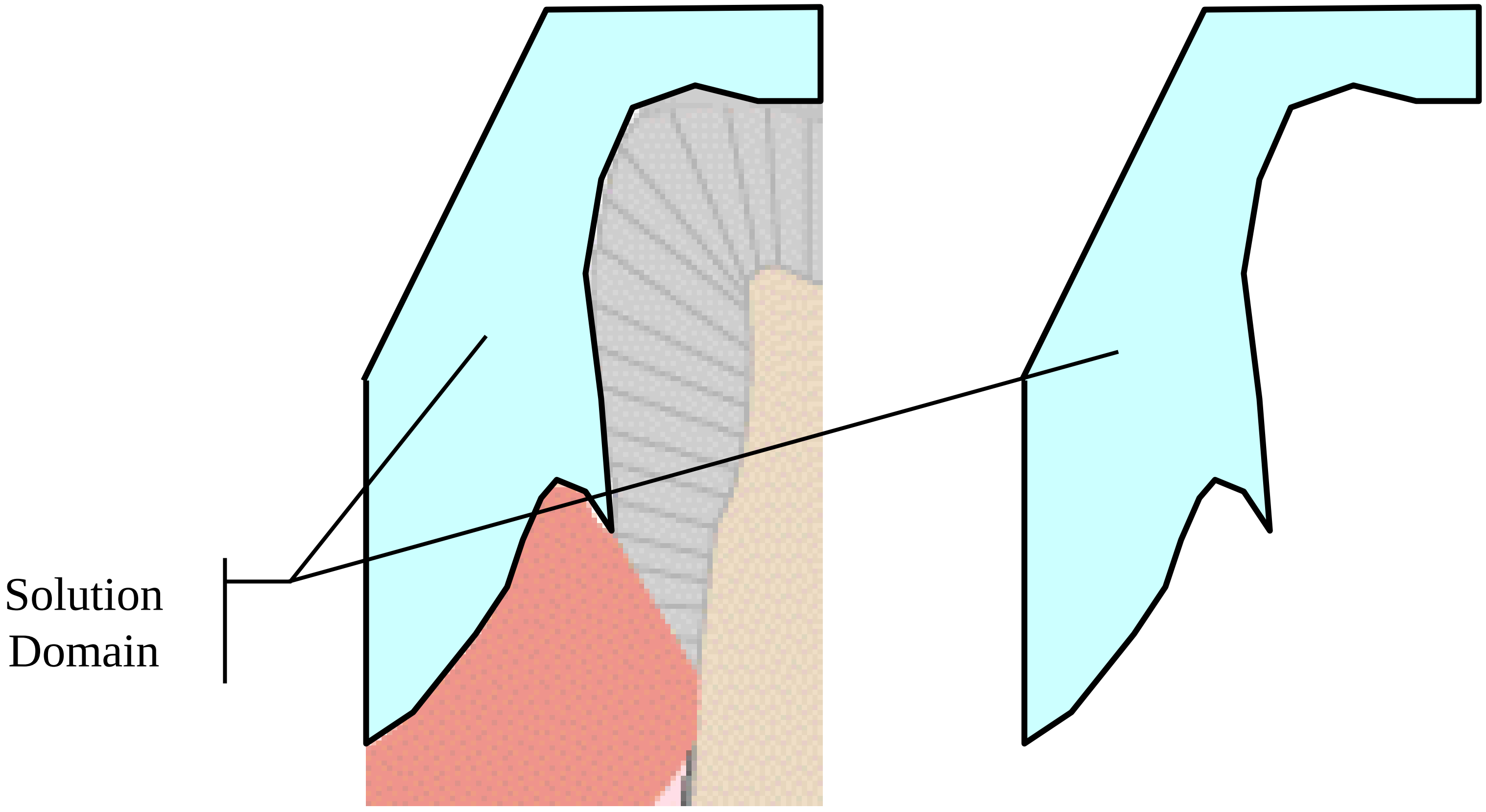
Part of a Tooth and Gum Line is Even Better



Solution Method

Solution Space Fluid Physics Equation Set

The incompressible Navier-Stokes vector-form equation is a nonlinear partial differential equation of second order as follows:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

with $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$ (Continuity – Conservation of Mass)

where v is a vector representing the velocity of an infinitesimal element of mass at a point in 3-D space, *p* is the scalar pressure at the same point, rho is the mass density at the point and is assumed constant throughout the medium, mu is the dynamic viscosity of the medium, and g is a constant vector acceleration due to some constant external force on the infinitesimal element, usually taken to be gravity.

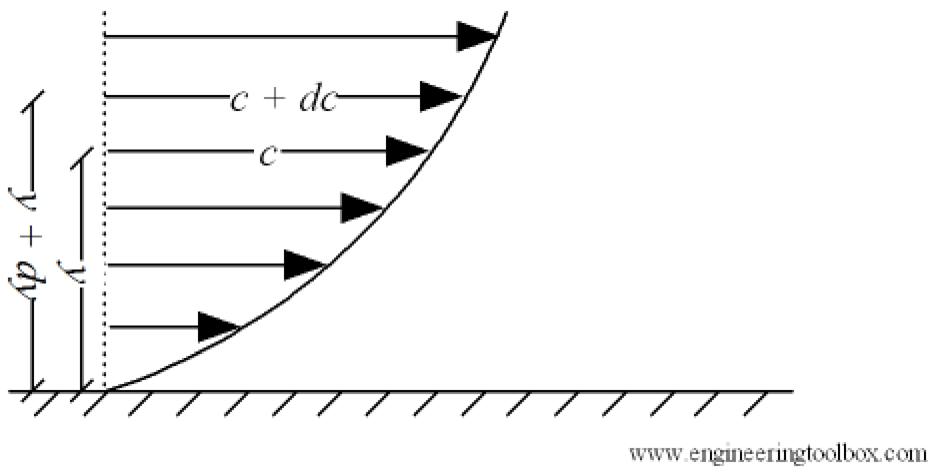
I used FlexPDE, a general purpose partial differential equation solving software engine, with appropriate boundary conditions, to solve the equation set.

Regarding Viscous Shear Forces

Dynamic (absolute) Viscosity

is the tangential force per unit area required to move one horizontal plane with respect to the other at unit velocity when maintained a unit distance apart by the fluid.

The shearing stress between the layers of non turbulent fluid moving in straight parallel lines can be defined for a Newtonian fluid as:



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Shear Force Laws for Non-Newtonian Fluids

Bingham fluids

In Bingham fluids, we have something slightly different: $\tau_{ij} = \tau_0 + \mu \frac{\partial v_i}{\partial x_j}, \ \frac{\partial v_i}{\partial x_j} > 0$

Those are fluids capable of bearing some shear before they start flowing. Some common examples are toothpaste and silly putty.

Power-law fluid

It is an idealized fluid for which the shear stress, τ, is given by $\tau = K \left(\frac{\partial u}{\partial y} \right)^n$

This form is useful for approximating all sorts of general fluids.

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Shear Force Measures and Units

The dynamic or absolute viscosity can be expressed like

$\tau = \mu \, dc/dy \quad (1)$

where

τ = *shearing stress*

μ = *dynamic viscosity*

Equation (1) is known as the **Newtons Law of Friction**.

In the SI system the dynamic viscosity units are **N s/m2**, **Pa s** or **kg/m s** where

1 Pa s = 1 N s/m2 = 1 kg/m s

The dynamic viscosity is also often expressed in the metric CGS (centimeter-gram-second) system as **g/cm.s**, **dyne.s/cm2** or **poise (p)** where

1 poise = dyne s/cm2 = g/cm s = 1/10 Pa s

For practical use the Poise is to large and it's usual divided by 100 into the smaller unit called the **centiPoise (cP)** where

1 p = 100 cP

Water at 68.4oF (20.2oC) has an absolute viscosity of one - 1 - centiPoise.

Fluid Media Defining Parameters Used in the Numerical Experiment

the dynamic viscosity, has values of,

for water 1 * 10 ^ -3 Newton*Seconds / m^2

for air 1.85 * 10 ^ -5 Newton*Seconds / m^2

(note that the dynamic viscosity for air is only 50 x less than water!).

Density of water is 10^3 kg/m^3

Density of air is 1.25 kg/m^3

(note that the density for air is 800 x less than water).

Air values are given for illustration – water was the media for this experiment

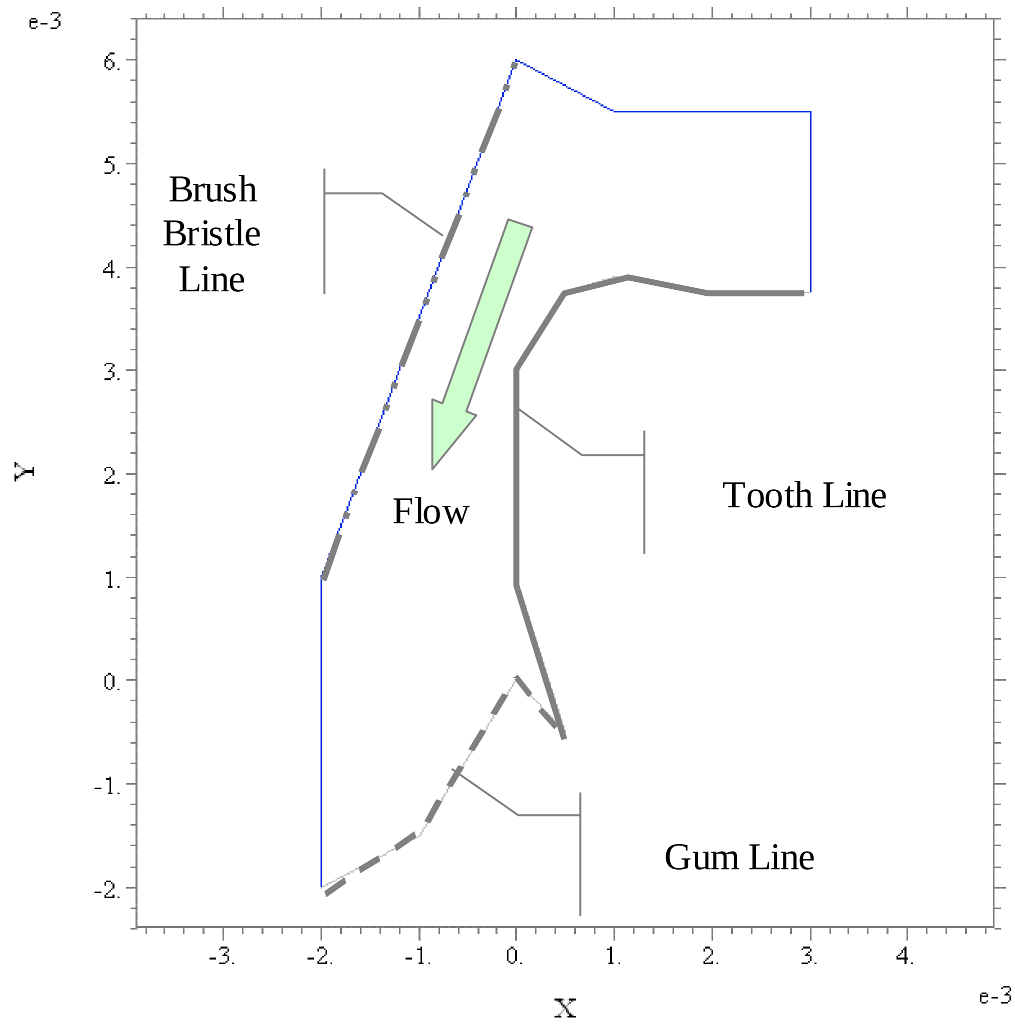
Analysis Domain and Experiment Parameters

Flow Near a Tooth & Gumline

Various Model Parameters

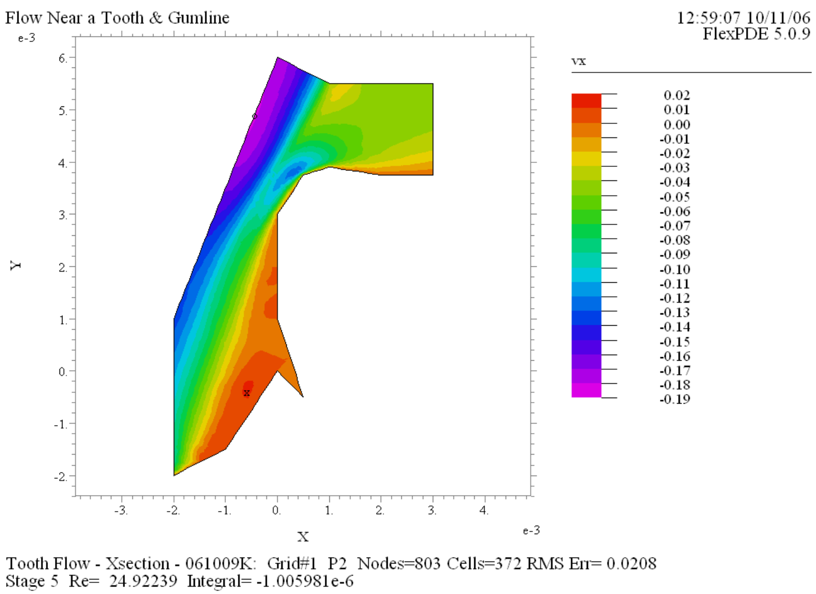
x1= -2.000000e-3
x2= -1.000000e-3
x3= 5.000000e-4
x4= 2.000000e-3
x5= 3.000000e-3
y1= -2.000000e-3
y2= -3.000000e-3
y3= -2.000000e-3
y4= -5.000000e-4
y5= 1.000000e-3
y6= 4.000000e-3
y7= 6.000000e-3
Dens= 1000.000
Visc= 1.000000e-3

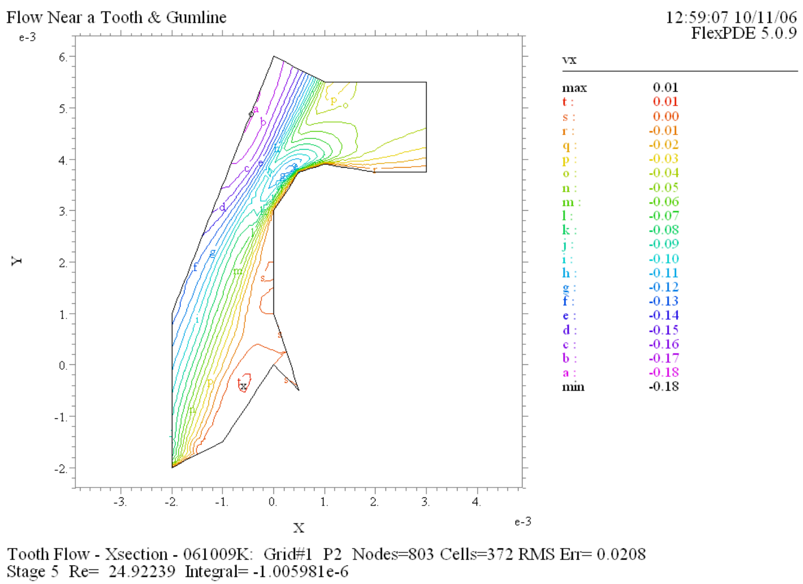
All units are MKS



Results

Velocity – X Direction





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Contour

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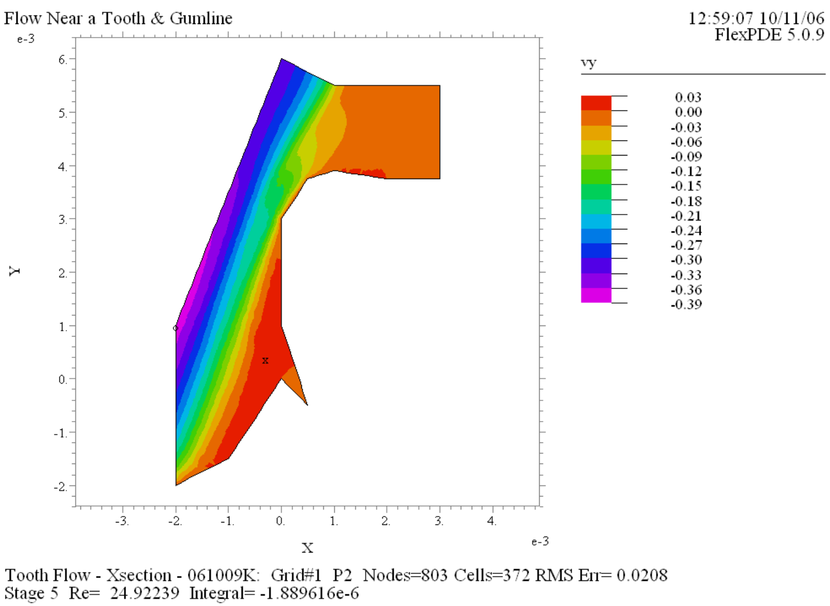
Velocity – y Direction



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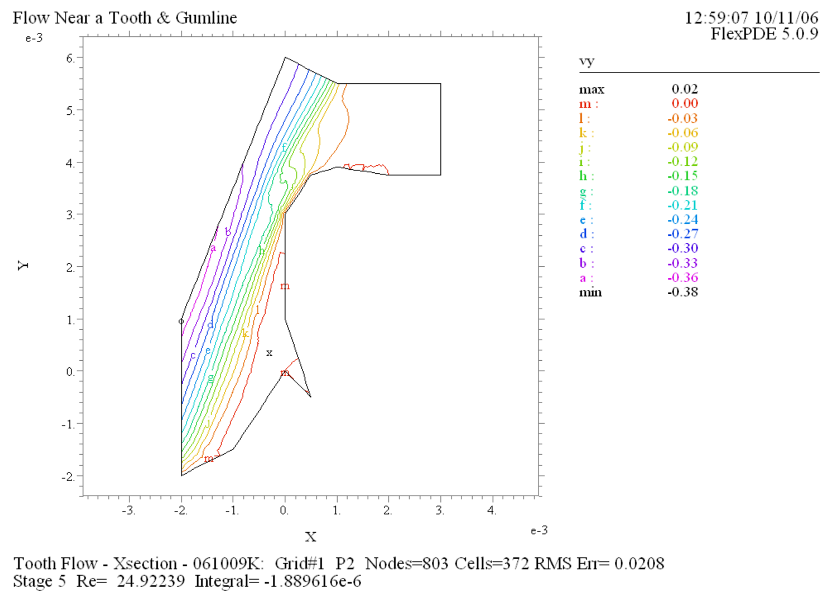

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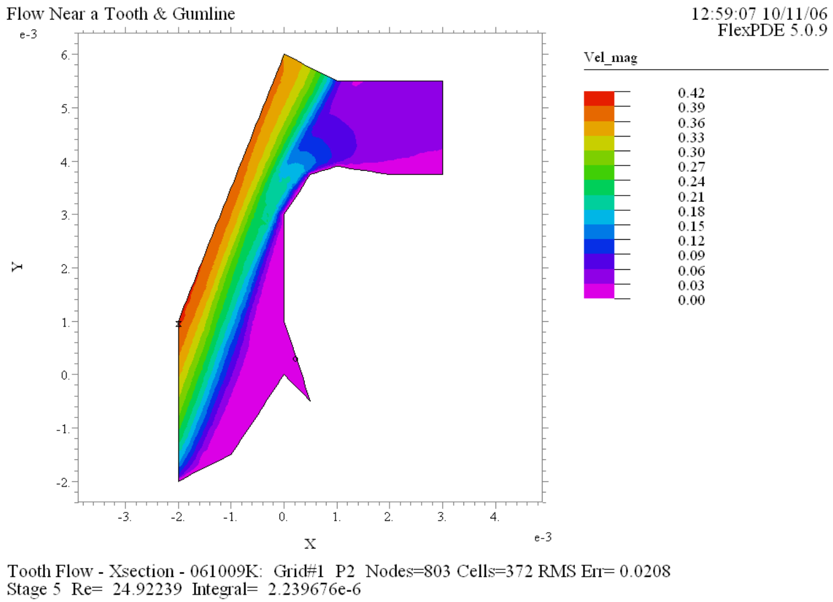
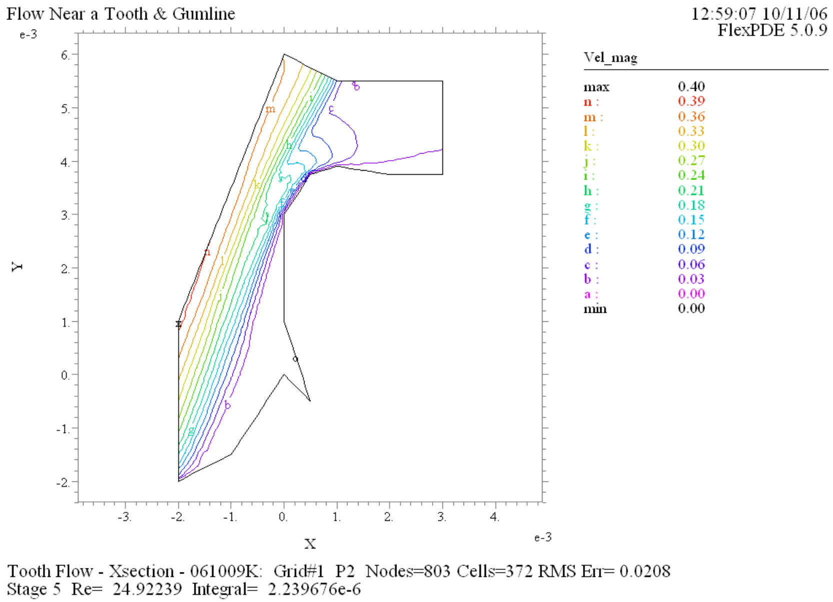
Velocity – Magnitude





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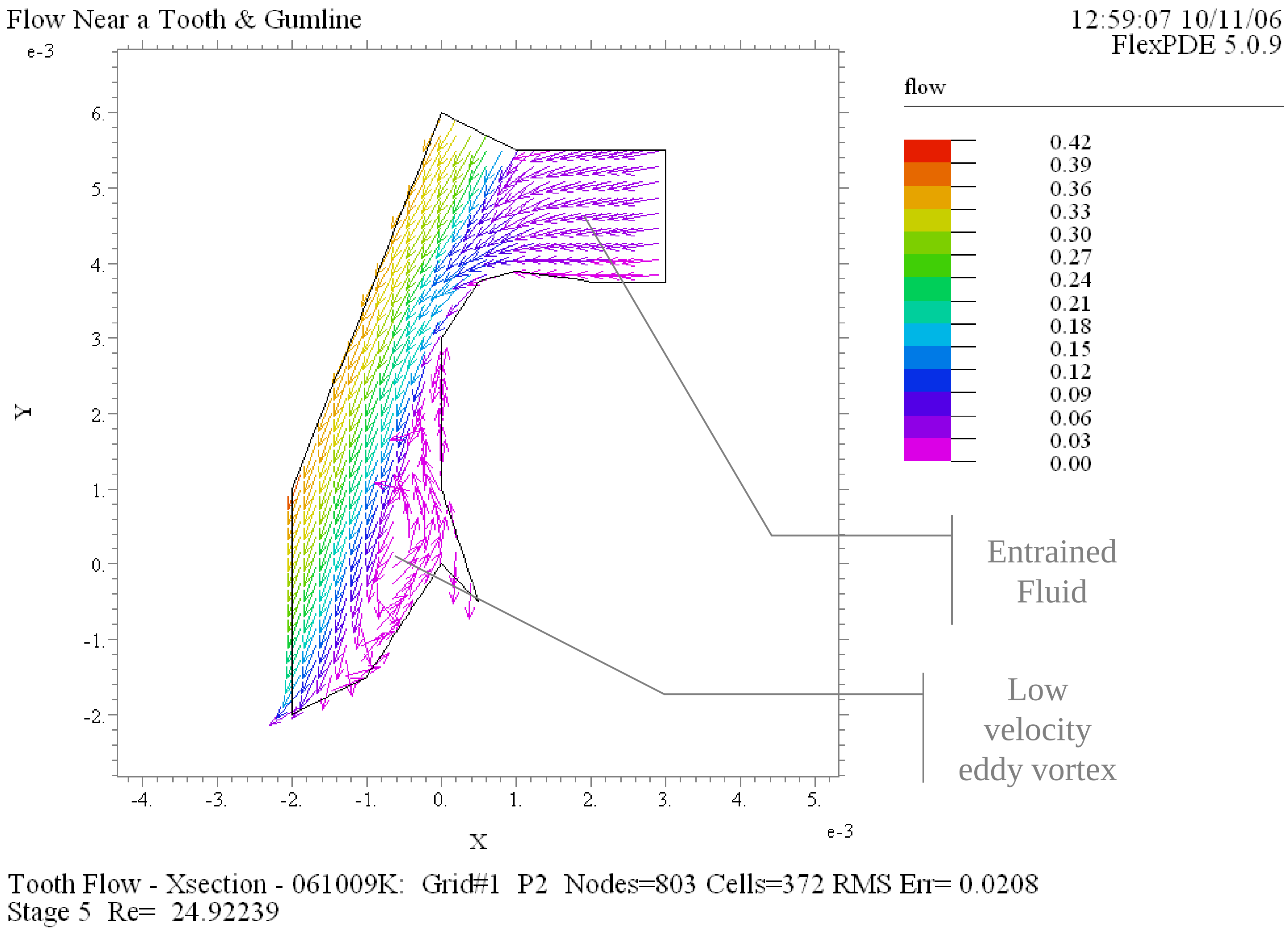
Velocity – Vector Plot



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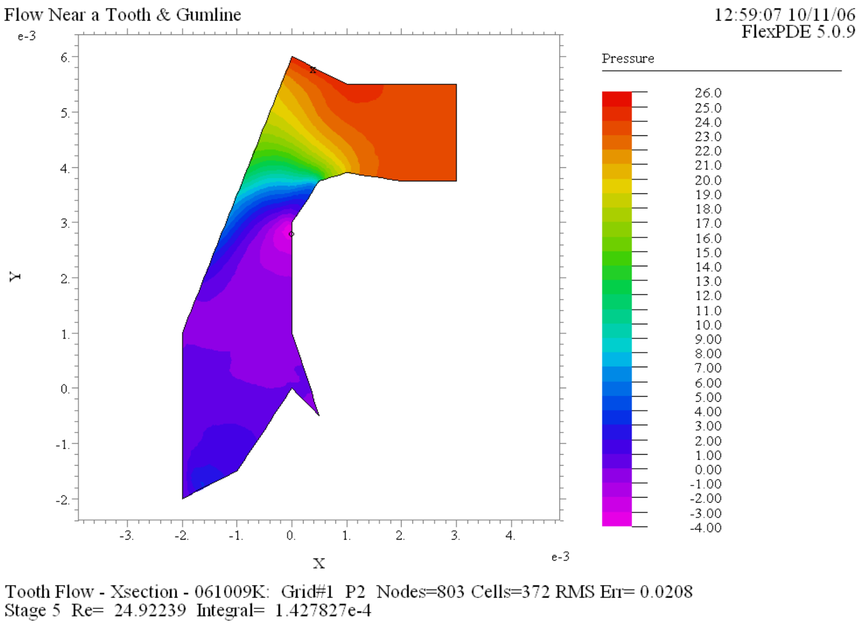
Pressure



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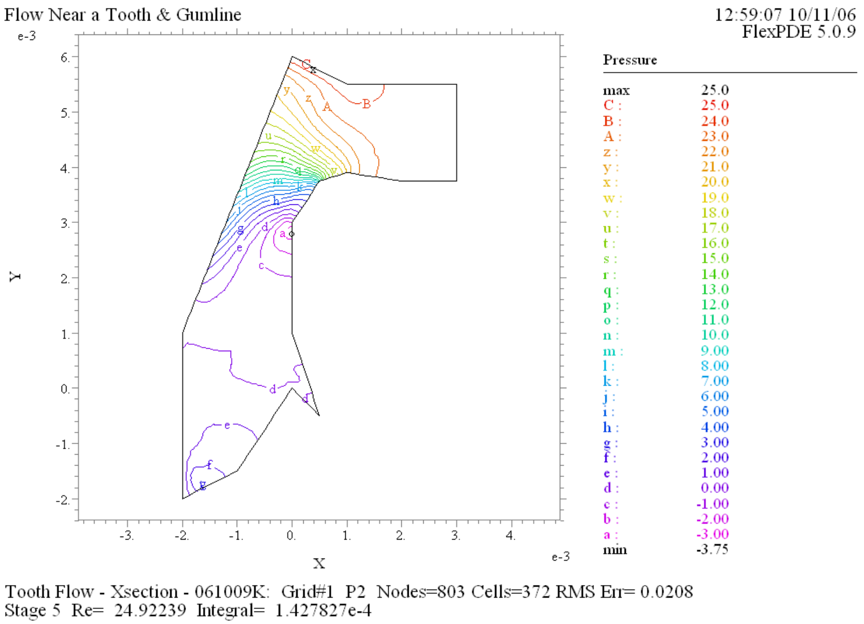

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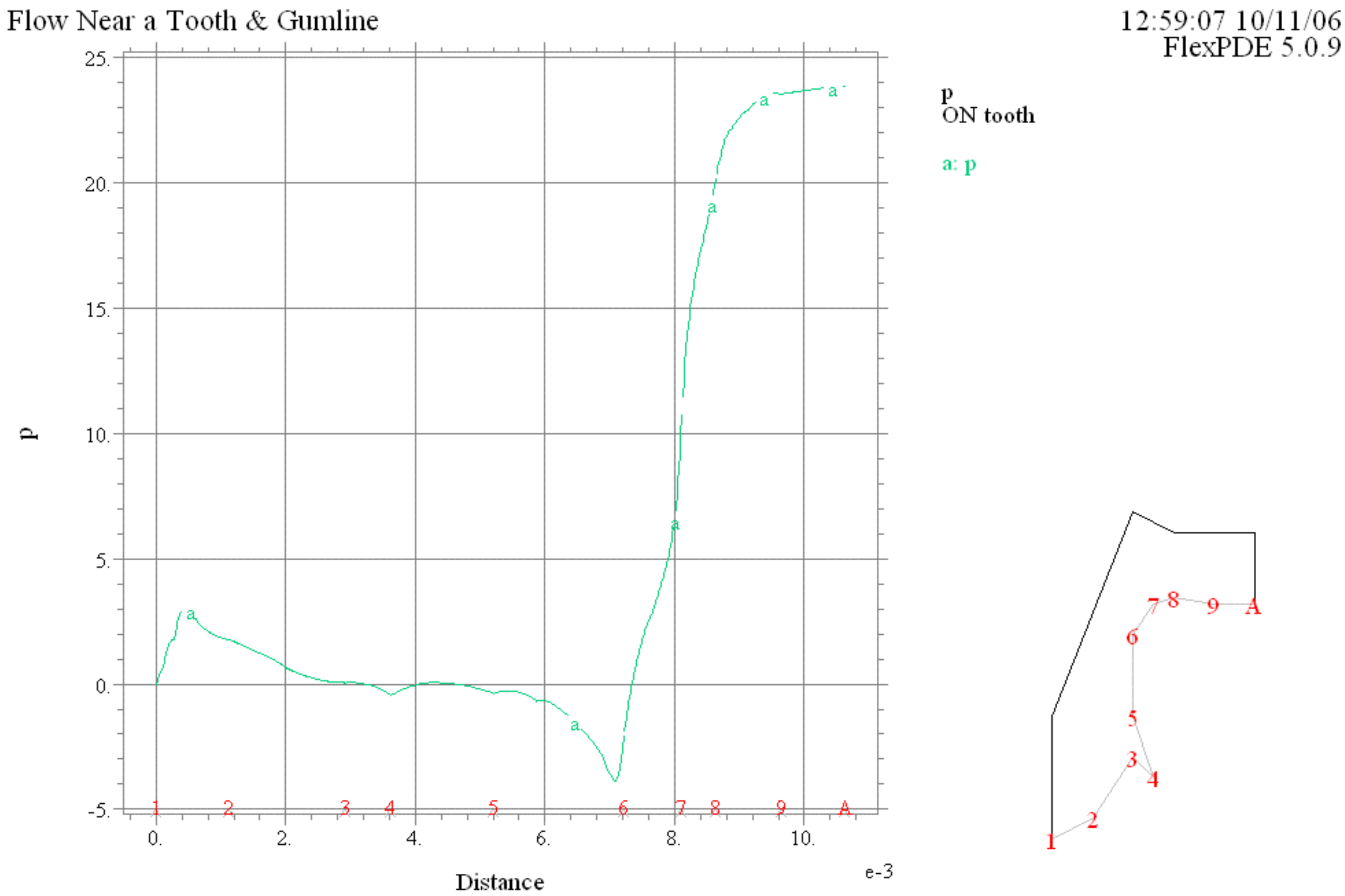
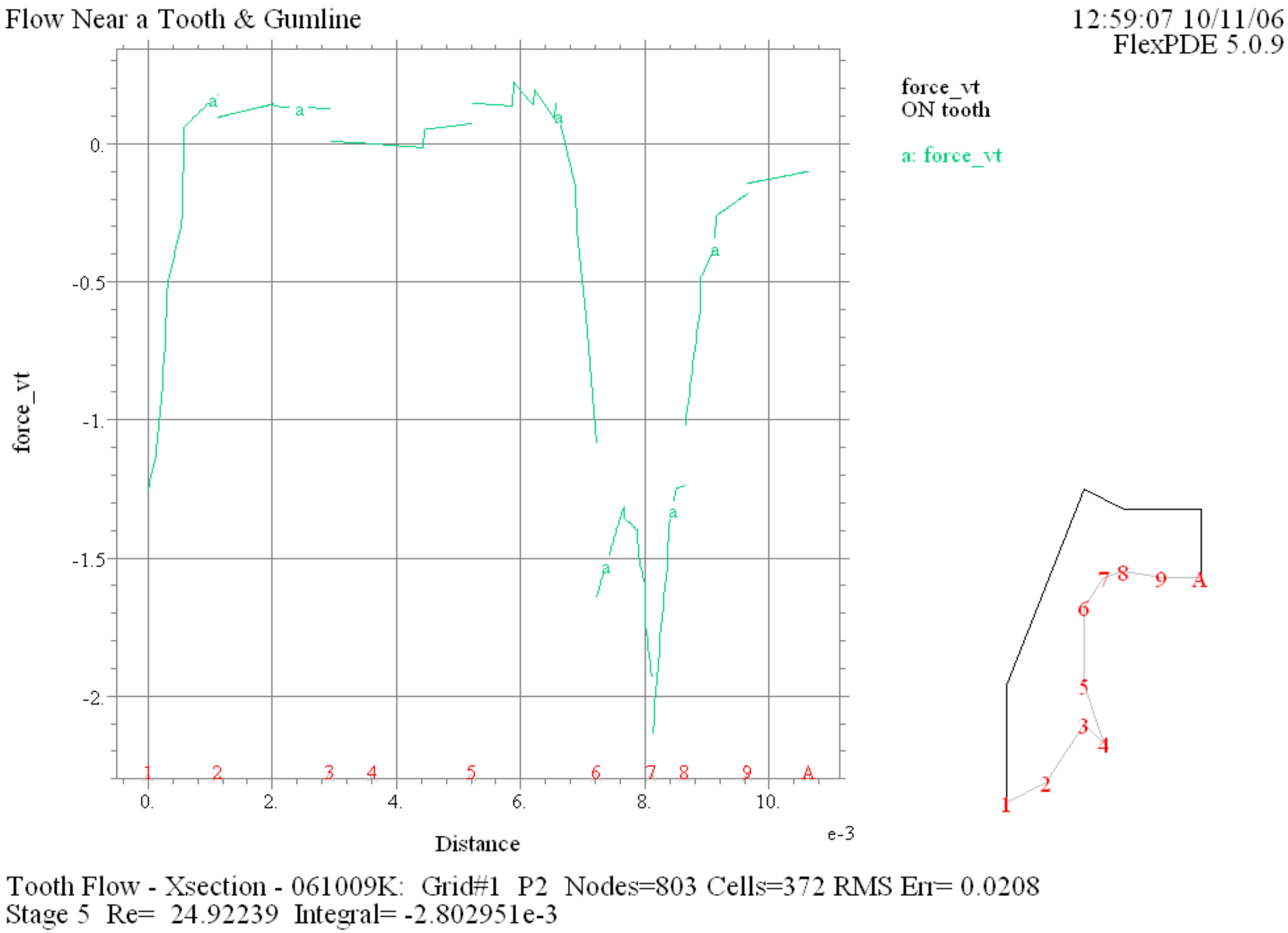
Pressure on Tooth and Gumline

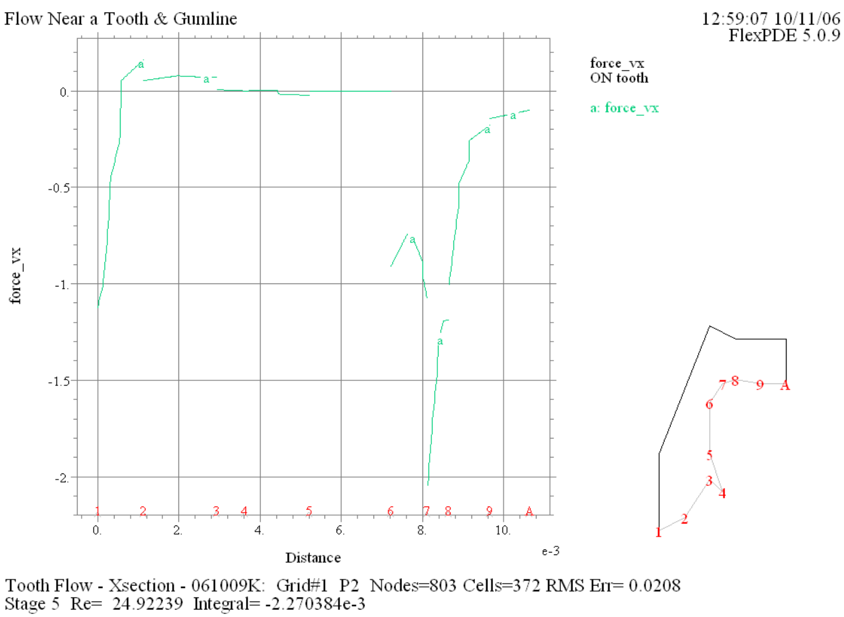


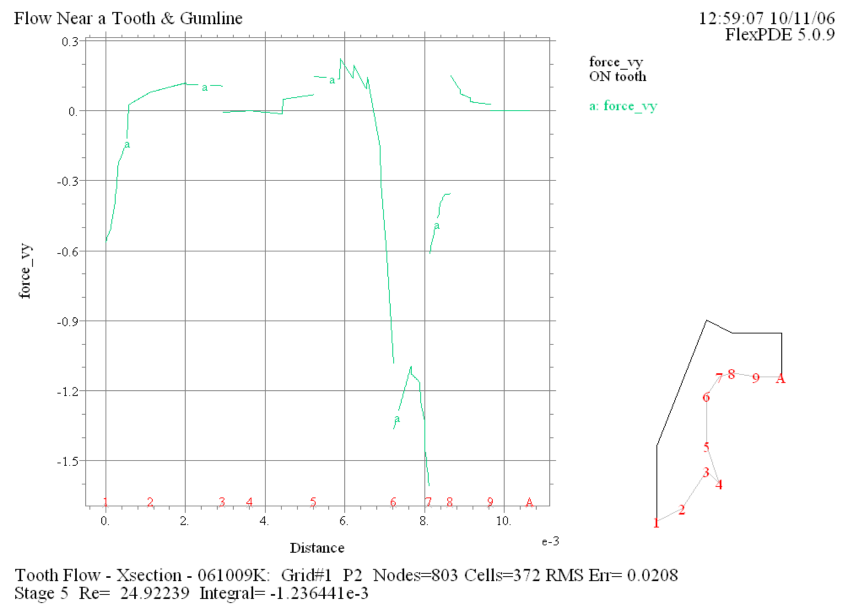
Tooth Flow - Xsection - 061009K: Grid#1 P2 Nodes=803 Cells=372 RMS Err= 0.0208
Stage 5 Re= 24.92239 Integral= 0.058317

Tangential Direction Velocity (Shear) Force



X and Y Direction Velocity (Shear) Force





X Direction

Y Direction

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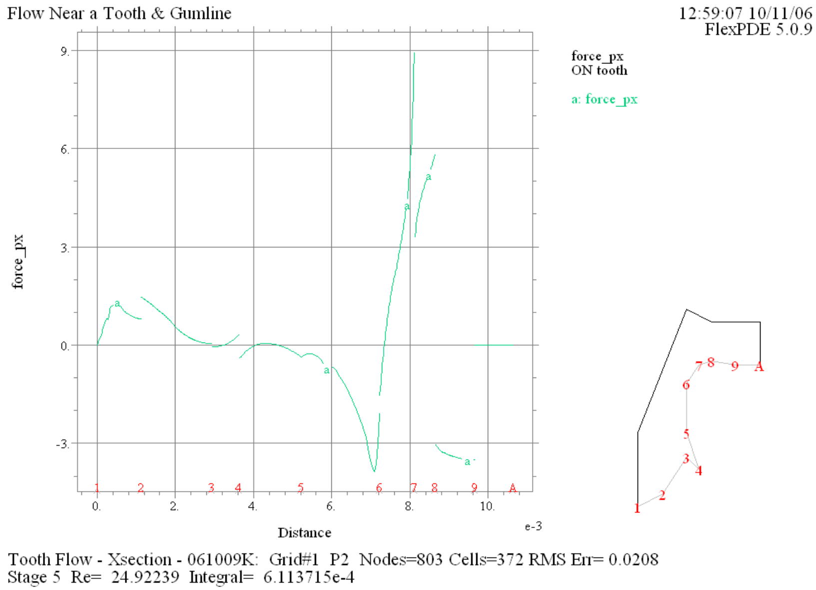
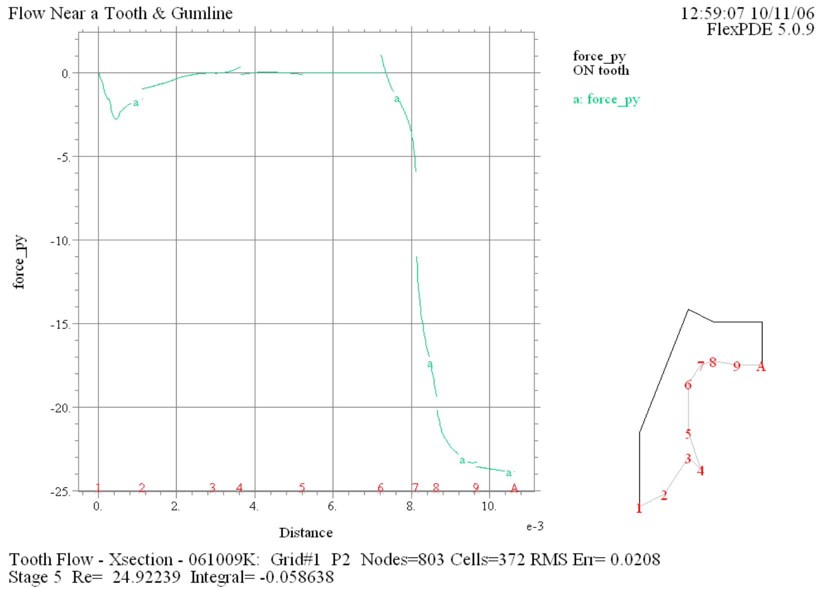
X and Y Direction Pressure Force

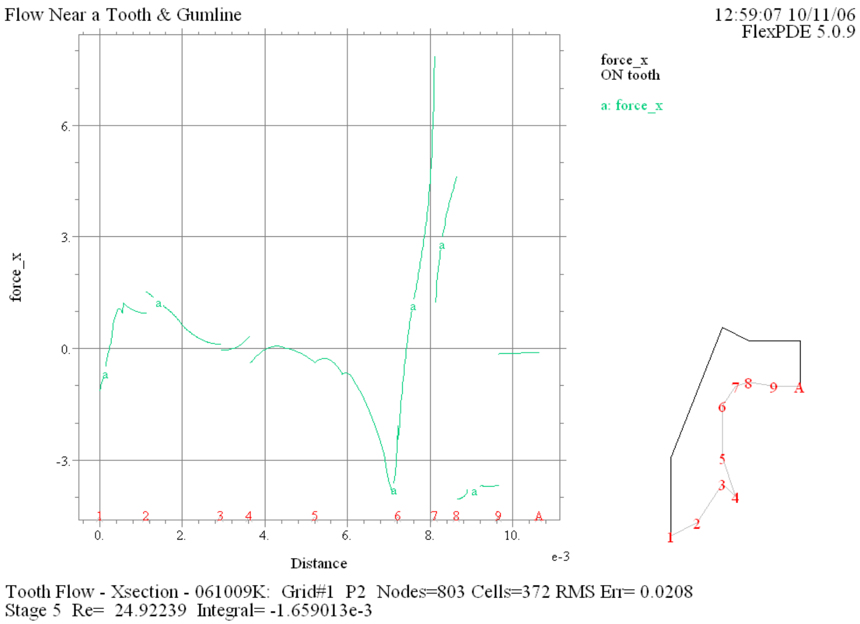


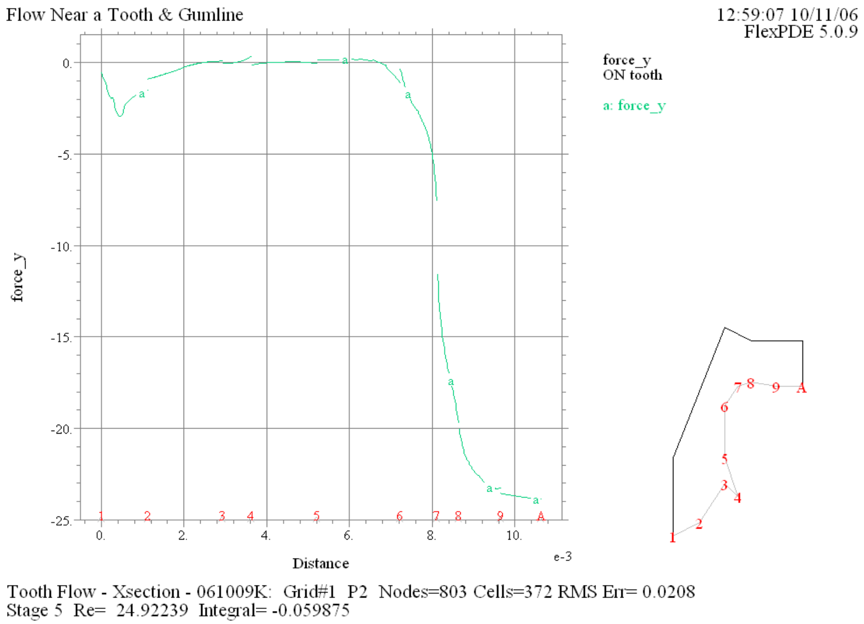


X Direction

Y Direction

X and Y Direction Total (Shear + Pressure) Force





X Direction

Y Direction

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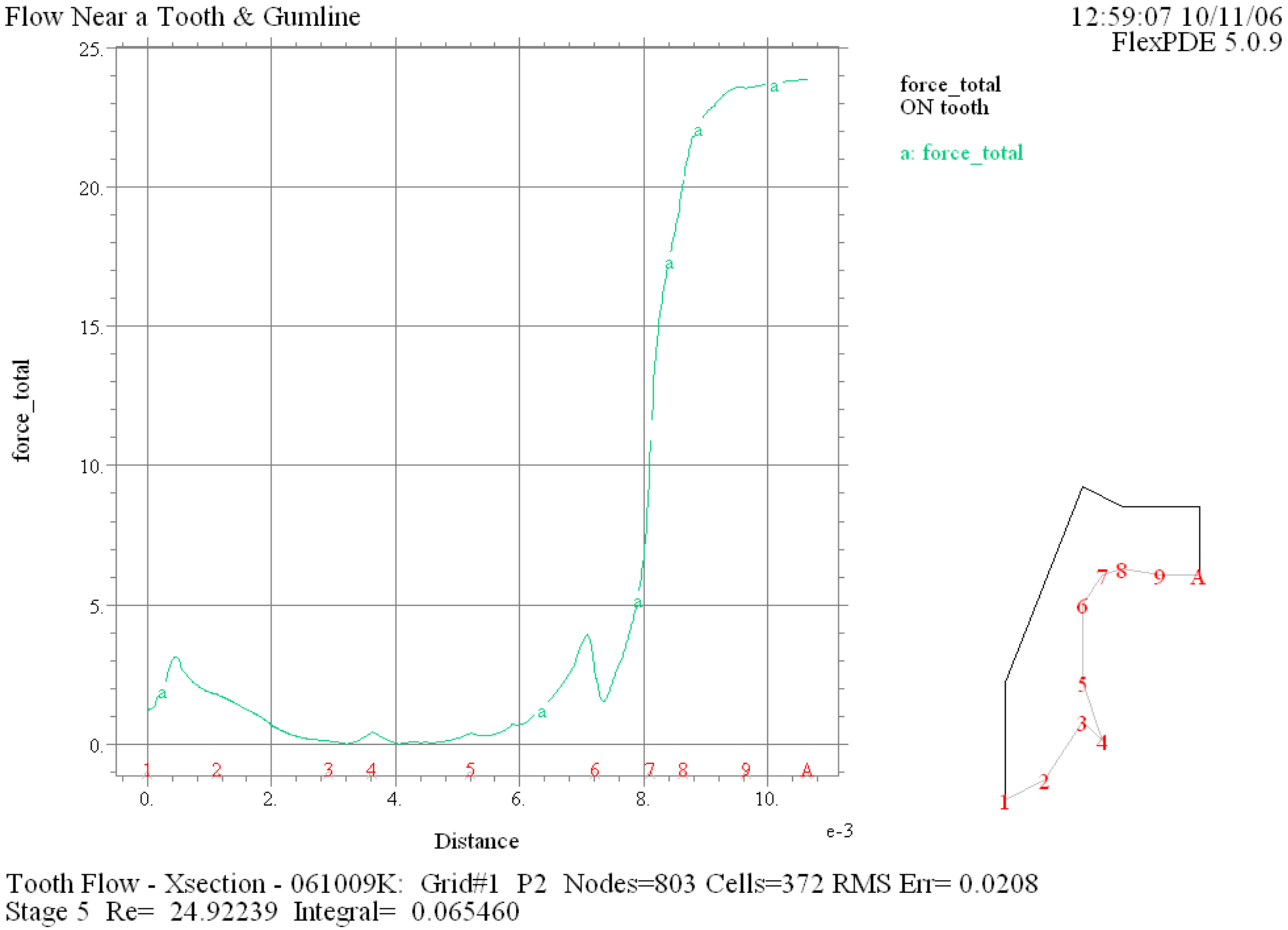
Total Force (Shear + Pressure) - Tooth and Gumline



Summary and Conclusions

1. Fluid flow near a tooth and gum line has been examined by means of a numerical experiment

2. The numerical experiment shows flow velocities and pressures that are reasonable to expect when a power tooth brush is used

3. Many other interesting and useful numerical experiments can be performed

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